

THESIS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

**A Study of Subsonic
Turbulent Jets and Their
Radiated Sound Using
Large-Eddy Simulation**

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Abstract

Stricter noise regulation for near-ground operations has made noise reduction in commercial aircraft a topic of growing interest in the aerospace industry. To meet airworthiness requirements new noise reduction technologies have to be developed and numerical methods for correct assessment of these technologies are desirable.

This thesis deals with predictions of near-field flow and far-field acoustic signature of subsonic turbulent single-stream and dual-stream jets at isothermal and heated conditions. The flowfield predictions are obtained using large-eddy simulation (LES), and Kirchhoff's surface integration technique is used to extend the acoustic domain to far-field locations. In all cases studied, the nozzle geometry is included in the calculation domain.

For the single-stream jet, predicted near-field flow statistics and far-field sound pressure levels (SPL) are both in good agreement with experiments. Predicted SPL for all observer locations, where evaluated, are within a deviation of 3.0 dB from measured levels and for most locations within a deviation of 1.0 dB. For the specific cases studied, Mach 0.75 jets, only small differences in radiated sound could be identified between an isothermal jet and a jet with temperature twice that of the surrounding fluid. The effects of changes in inflow conditions, Reynolds number and subgrid-scale (SGS) model on the flowfield and acoustic signature were investigated. Only minor changes could be identified in the predictions of flow statistics and radiated sound.

For the dual-stream jet, changing the subgrid-scale filter width and introducing a TVD limiter gave significant changes in the shear layer flow. Sound radiated in the upstream direction was shown to depend appreciably on the initial shear layer development. Vortex generators placed on the outside wall of the inner nozzle were found to effectively break up ring-shaped vortical structures in the initial inner shear layer region and speed up the mixing between the core and bypass streams.

Keywords: CAA, Aeroacoustics, Jet Noise, LES, Kirchhoff, Heated Jets, Coaxial Jets, Two-Point Space-Time Correlations

List of Publications

This thesis is based on the work contained in the following papers:

- I N. Andersson, L.-E. Eriksson and L. Davidson, 2005, Investigation of an Isothermal Mach 0.75 Jet and its Radiated Sound Using Large-Eddy Simulation and Kirchhoff Surface Integration, *International Journal of Heat and Fluid Flow*, 26(3), 393–410
- II N. Andersson, L.-E. Eriksson and L. Davidson, 2005, Large-Eddy Simulation of Subsonic Turbulent Jets and Their Radiated Sound, *AIAA Journal* 43(9) 1899–1912
- III N. Andersson, L.-E. Eriksson and L. Davidson, 2005, Effects of Inflow Conditions and Subgrid Model on LES for Turbulent Jets, *The 11th AIAA / CEAS Aeroacoustics Conference*, AIAA 2005-2925, May 23-25, Monterey, California
- IV N. Andersson, L.-E. Eriksson and L. Davidson, 2005, LES Prediction of Flow an Acoustic Field of a Coaxial Jet, *The 11th AIAA / CEAS Aeroacoustics Conference*, AIAA 2005-2884, May 23-25, Monterey, California

Division of Work Between Authors of the Papers

The work leading up to this thesis was done in collaboration with other researchers. The respondent is the first author of all papers on which this thesis is based, and the respondent produced all results. Theoretical work and code development presented in the papers were carried out in discussions with supervisors Lars-Erik Eriksson and Lars Davidson.

Other Relevant Publications

N. Andersson, L.-E. Eriksson and L. Davidson, 2003, Large-Eddy Simulation of a Mach 0.75 Jet, *The 9th AIAA/CEAS Aeroacoustics Conference*, AIAA 2003-3312, May 12-14, Hilton Head, South Carolina

N. Andersson, 2003, A Study of Mach 0.75 Jets and Their Radiated Sound Using Large-Eddy Simulation, Licentiate thesis, *Division of Thermo and Fluid Dynamics, Chalmers University of Technology, Gothenburg*

N. Andersson, L.-E. Eriksson and L. Davidson, 2004, A Study of Mach 0.75 Jets and Their Radiated Sound Using Large-Eddy Simulation, *The 10th AIAA/CEAS Aeroacoustics Conference*, AIAA 2004-3024, May 10-12, Manchester, United Kingdom

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I would like to acknowledge Mattias Billson, Johan Larsson and Jonas Ask who together with me, with great support from Lars-Erik Eriksson and Lars Davidson, initiated the computational aeroacoustics research activities at our department. Sharing experiences, setbacks and successes, we have solved problems together that would have been tough to deal with alone. This study would definitely not have reached this stage without the help from you guys. Special thanks to Mattias Billson for all helpful discussions and encouraging talks. Moreover, our joint code development effort is something that I look back upon as the most fun part of my time as a graduate student.

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To Caroline

Nomenclature

Latin symbols

c	speed of sound
C_p	specific heat at constant pressure
C_v	specific heat at constant volume
C_R, C_I	Smagorinsky model coefficients
D_j	nozzle outlet diameter
e	energy
$\mathcal{F}_j$	flux component
f	frequency
k	kinetic energy
L_c	potential core length
p	pressure
Pr	Prandtl number
q_j	energy diffusion vector
Q	state vector in equations on conservative form
q	state vector in equations on primitive form
$\mathcal{Q}$	cell volume averaged state vector
R	gas constant
r	radial coordinate or distance from source to observer
Re_D	Reynolds number based on the jet diameter
S_{ij}	strain rate tensor
$\mathcal{S}_j$	cell face area normal vector
St	Strouhal number ($St = (fD_j)/U_j$)
T	temperature
t	time
T_{ij}	Lighthill stress tensor
(u, v, w)	axial, radial and tangential velocity component
u_i	Cartesian components of velocity vector
U_j	jet-exit velocity
V	volume
$w_1 - w_5$	characteristic variables
$\mathbf{x}$	flowfield location
x_i	Cartesian coordinate vector component

y far-field observer location

Greek symbols

Δ filter width
 δ_{ij} Kronecker delta
 $\lambda_1 - \lambda_5$ characteristic speeds
 μ dynamic viscosity
 ν kinematic viscosity ($\nu = \mu/\rho$)
 ρ density
 σ_{ij} viscous stress tensor
 τ_{ij} subgrid-scale stress tensor
 τ_r retarded time
 ε_ν viscous dissipation
 ξ_i computational space coordinate vector component

Subscripts

0 total condition
 ∞ freestream or ambient conditions
 j jet, nozzle-exit condition
 t turbulent quantity

Superscripts

— spatially filtered quantity
 $'$ resolved fluctuation
 $''$ unresolved quantity
 $\sim$ spatially Favre-filtered quantity
 SGS subgrid scale

Abbreviations

CAA Computational Aero Acoustics
CFD Computational Fluid Dynamics
CFL Courant-Friedrichs-Lewy
DNS Direct Numerical Simulation
HBR High Bypass Ratio
LDV Laser Doppler Velocimetry
LEE Linearized Euler Equations
LES Large Eddy Simulation
MPI Message Passing Interface
RANS Reynolds-Averaged Navier-Stokes
SGS Subgrid Scale
SPL Sound Pressure Level
TVD Total Variation Diminishing

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Chapter 1

Introduction

1.1 Motivation

THE number of commercial aircraft in service is continuously growing, and airports around the world are growing in size, which increases exposure to air traffic noise in populated areas. Threshold values for noise certification of new aircraft are based on global restrictions on noise generated by air traffic. Moreover, local restrictions for airports with heavy traffic limit operating hours or even impose direct noise penalty costs. Stricter regulations on noise levels in the surroundings of airports have made the abatement of near-ground operation noise an important issue for aircraft and engine manufacturers, and noise generation has now become an important design factor that is taken into consideration early in the construction process. Despite progress in the development of computational fluid dynamics (CFD) solvers, most of the noise prediction methods currently in use in industry are correlations based on empirical databases. The reason for this is the extreme demands for numerical accuracy in computational aeroacoustic (CAA) methods that place high demands on computational resources. However, to be able to account for changes in the flow by new noise reduction techniques or to predict differences in radiated sound in different engine concepts at an early design stage, more explicit approaches are required. With continuously increasing computer capacity and with the possibility to carry out parallel computations on PC clusters, computational aeroacoustics has now, to some extent, become feasible for industrial use.

Flow-induced aircraft noise can be divided into two categories: airframe noise and noise generated by the jet engine. The first category includes noise generated by landing gear, high-lift devices and the aircraft fuselage itself and the second includes turbo-machinery noise,

core noise and jet noise. At take-off, the main sources of noise are the propelling jet and the engine fan, of which the jet exhaust is usually the strongest noise source at full power. In commercial aircraft, increasing the bypass ratio, i.e. the ratio of mass passing the engine in the bypass duct to the mass passing through the engine core, has given a significant reduction in aircraft noise since the 1960s. However, although an increase of the bypass ratio leads to lower noise levels, the major motivation for this development has not been noise reduction. Rather, the development towards more efficient engines has led to the use of a higher bypass ratio with noise reduction as a positive side effect. The lower noise levels of high-bypass ratio (HBR) engines are directly attributable to the reduction in jet noise resulting from lower jet velocities. Unfortunately, without a step change in technology, the maximum bypass ratio is limited by a number of factors, e.g. the length of the fan blades, engine weight, rotor speed and engine nacelle drag, and large engines are currently very close to this limit. Consequently, the possibility for reducing jet noise by increasing the bypass ratio is rapidly decreasing.

Other techniques to lower noise levels have been investigated in the past decades. Among these concepts, many are of a mixing enhancement nature, e.g. chevrons, lobed mixers and tabs. The lobed mixer efficiently evens out differences in the velocity of the core flow and the bypass flow, which reduces the exhaust velocity and hence the sound generated. Chevrons and tabs are both devices added to the nozzle geometry that protrude into the flow and generate axial vorticity and thereby enhance the mixing of core, fan and ambient air streams. While these noise-reducing concepts have proved to be able to lower noise levels (e.g. Saiyed *et al.*, 2000; Nesbitt *et al.*, 2002), the reduction comes with a penalty on efficiency. The contradiction of noise reduction for near-ground operation and requirements for higher thrust and engine efficiency at cruise conditions will probably be common in all new noise-reducing concepts. Furthermore, requirements for higher thrust are often satisfied by increasing the flow through existing engines with only minor modifications, which leads to higher exhaust velocities and temperatures and increases the contribution of the jet to the overall noise.

Part of the work presented in this thesis has been conducted within the EU 5th framework programme JEAN¹, which was an European project investigating the physics behind noise generation in isothermal and heated single-stream jets. In this project, jet noise mechanisms

¹JEAN – Jet Exhaust Aerodynamics & Noise, Contract number: G4RD-CT-2000-000313

were investigated both numerically and experimentally and various known methodologies for noise prediction were tested and compared for a few test cases. In the continuation of that project, CoJeN², the flow-field and radiated sound of a high-subsonic coaxial nozzle/jet configuration is being investigated. The work reported in Paper IV was done within the framework of the latter of the two projects. The objective of CoJeN is to develop and validate prediction tools that can be used by the aerospace industry to assess and optimize jet noise prediction techniques. The aim is to provide design tools that can be used to develop low-noise nozzles for HBR engines. For prediction techniques to be useful to industry, the methods must cope with realistic jet flows such as coaxial jet configurations with high velocities, significant velocity and temperature gradients, and arbitrary nozzle geometries. Hence, the ability to capture the initial flow physics becomes very important. LES is not currently feasible for industrial design use because of long turnover times and a restriction to fairly simple geometries. However, new noise-reducing concepts will arise from a better understanding of the source mechanisms. To evaluate the performance of these new concepts, reliable methods for modeling the source mechanisms must be available. The results of a detailed LES can be used to gain a more realistic picture of the flow physics and thus be useful in the development of tools for industrial use. For example, higher-order statistics important for the understanding of noise generation, such as two-point space-time correlations, can be evaluated. Detailed information on the statistical character of the flow can later be used to develop prediction techniques based on noise generation mechanisms. This is the main objective of the present work.

1.2 Turbulent Free Jet

The turbulent free jet is an example of a free shear flow, i.e. a flow with mean flow gradients that develop in the absence of boundaries (George, 2000). This kind of flow is characterized by a main flow direction in which the velocity is significantly greater than in the transverse direction. The gradients in the transversal direction are also much larger than those in the main direction. These kinds of flows are common both in nature and in engineering applications. Examples are the residual gases spread into the atmosphere by a furnace chimney and the propelling jet of an aircraft engine. The turbulent jet is separated from its non-turbulent surroundings by an interface often referred to

²CoJeN – Computation of Coaxial Jet Noise, contract number: AST3-CT-2003-502790.

as the viscous super layer, by analogy with the viscous sublayer of a boundary layer, or the Corrsin super layer after its discoverer (Hinze, 1975; George, 2000). The shape of the interface is random and continuously changing and its thickness is characterized by the Kolmogorov microscale (George, 2000). The flow in these outer regions of the jet is therefore intermittent in nature, i.e. sometimes turbulent and sometimes not, as the shape of the jet and its interface with the surroundings changes. The level of intermittency increases radially outwards through the jet shear layer. Minimum intermittency is found in the radial location in which the shear is highest (Wynanski & Fiedler, 1969). The shape of the interface is strongly affected by the turbulent flow in the jet, and then mainly by the larger structures of the flow (Hinze, 1975; George, 2000). The motion of the interface induces irrotational fluid motion of the surrounding fluid. The amount of turbulent fluid continuously increases downstream due to entrainment, i.e. fluid is entrained to the turbulent jet from its surroundings. The entrainment process causes the jet to spread in the transversal direction, and the jet flow can therefore never reach homogeneity (George, 2000). As long as the jet remains turbulent, the range of scales present in it will increase as a result of its increasing size. In the entrainment process, mass is continuously added to the turbulent jet, but no momentum is added.

Figure (1.1) shows the development of a single-stream jet. High-velocity fluid is continuously added through a nozzle into stagnant surroundings. As it exits the nozzle, the flow of the high-velocity fluid is fully aligned with the nozzle wall, and a core region of potential flow is formed. A shear layer is generated between the high-velocity fluid and its surroundings. The thickness of this shear layer depends on the thickness of the boundary layer at the nozzle exit. Due to entrainment of ambient fluid, the shear layer grows in size downstream. As the width of the shear layer increases, the radial extent of the potential core region decreases and more and more of the flow becomes turbulent. Shortly after the potential core closure, the entire jet is turbulent and thus fully developed.

The jet becomes self-preserving or self-similar in the fully developed region. This means that profiles of mean flow quantities can be collapsed by proper scaling. It has long been assumed that these self-similar profiles are independent of initial conditions for all quantities and therefore universal for all jets. This assumption has been questioned by George (1989), however.

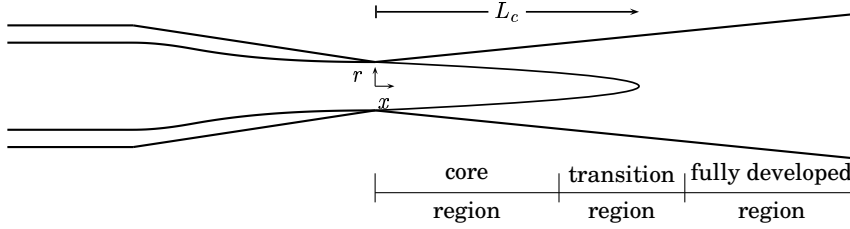


Figure 1.1: Flow regions in a developing jet

1.3 Jet Noise

Investigation of jet noise was more or less initiated in 1952 when Lighthill proposed his acoustic analogy in the first of his two-part paper on aerodynamically generated sound (Lighthill, 1952, 1953). These publications focused mainly on the sound generated by turbulent jets, but the acoustic analogy presented therein has been used extensively for numerous applications in the area of aeroacoustics. In addition to the work of Lighthill, a number of researchers such as Curle (1955); Lilly (1958); Ffowcs Williams (1963); Ffowcs Williams & Hawkings (1969); Ribner (1969) have made significant contributions to the theory and applications of acoustic analogies. A thorough review of the development of acoustic analogies can be found in Crighton (1975).

The understanding of flow-induced noise, for example jet noise, is strongly coupled to the understanding of turbulence since the sources of sound are defined by the turbulent flow itself. Fifty years ago, when jet noise research was initiated, turbulence was regarded as consisting of randomly distributed small eddies. Thus the research focused on the noise generated by fine-scale structures. The focus changed with the discovery of large turbulence structures in free shear flows in the early 1970s and it was believed that, for high-speed jets, the dominant part of the sound generated was generated by these large structures. More recent analysis has shown that both the fine scales and the larger structures are responsible for the noise that is generated (Tam, 1998).

In jet flows, the sound-generating structures are convected downstream by the mean flow. It can be shown that moving sources tend to radiate more sound in the direction in which the source is transported (Tam, 1998; Ribner, 1969). Moreover, the sound generated is affected by the mean convection such that when the wave front propagates through the jet on the way to the far-field region its path tends to bend away from the jet axis. The reason for this is that the con-

vection velocity is highest in the centerline regions (A) and lower in the outer jet regions (B), see Figure (1.2). This effect is referred to as refraction and creates a cone of relative silence downstream of the noise-generating region since less sound is radiated in this direction. The refraction effect is more noticeable for jets with a higher temperature than the surrounding fluid since the speed of sound then varies over the jet cross section (Tam, 1998).

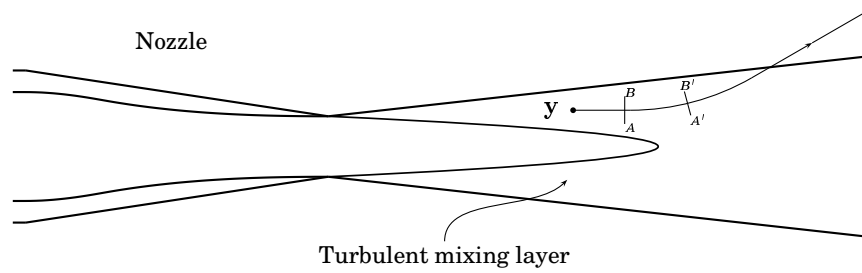


Figure 1.2: Refraction of a sound wave generated at position y in the shear layer and propagating through the jet. A wave front defined by line AB is propagating downstream at a velocity defined by the speed of sound and the local flow velocity. The flow velocity is higher in A than in B , which results in a tilting of the wave front towards line $A'B'$.

According to Ribner (1969), the sound generated by a turbulent jet can be separated into two parts. These are the sound generated by interaction of turbulence fluctuations and the sheared mean flow and the sound generated by turbulence fluctuations interacting with themselves; for greater detail see e.g. Ribner (1969). The two sound components are often referred to as *shear noise* and *self noise*, respectively. The *self noise* is radiated equally in all directions whereas the *shear noise* gives a dipole-like contribution. Superimposed, these two contributions give an ellipsoid-shaped sound radiation pattern, with the most sound radiated along the jet axis. Adding the effect of convection, the part of the sound that is radiated downstream of the jet is significantly larger than the part radiated in the upstream direction. Refraction significantly decreases the sound radiated along the jet axis. The contribution to the overall sound of the two components and the effect of convection and refraction on the pattern of radiated sound are depicted in Figure (1.3).

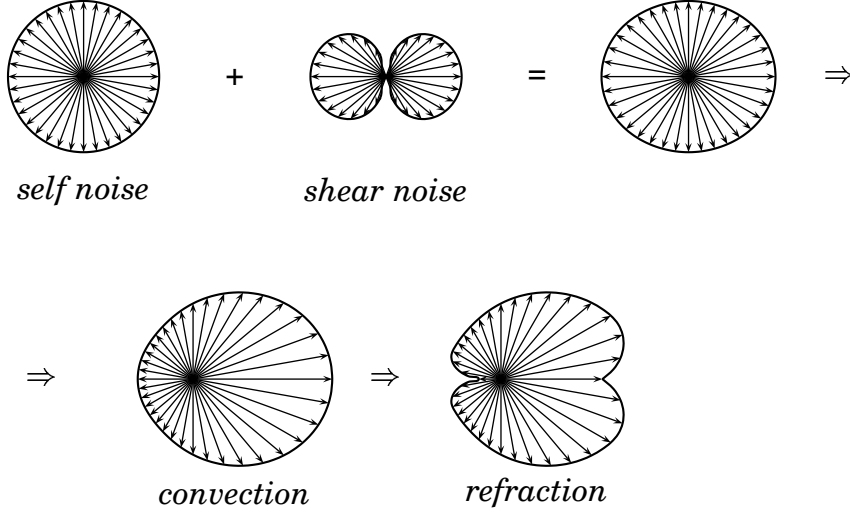


Figure 1.3: Superimposition of the noise generated by the turbulence itself, *self noise*, and the noise generated by interaction of the mean shear flow and turbulence, *shear noise*, gives the upper right sound pattern. Convection and refraction affect the pattern of radiated sound, and the result is the directional sound pattern in the lower right figure.

1.4 Computational Aeroacoustics

Using a grid fine enough in the far-field regions to minimize the introduction of sound propagation errors, the acoustic field can be obtained directly from the flowfield simulation. This requires a detailed numerical compressible flow simulation, e.g. direct numerical simulation (DNS), see for example Freund (2001) and Mitchell *et al.* (1999), or LES, as in e.g. Bogey *et al.* (2000a, 2003); Bogey & Bailly (2003, 2004, 2005b) and Mankbadi *et al.* (2000). In DNS, all scales of the turbulent flowfield are computed accurately, which requires a mesh fine enough to capture even the smallest scales of the flow, whereas in LES only the large scales of the flow are resolved and the influence on these large scales of the smaller, unresolved scales is modeled using a subgrid-scale model. With the computational resources available today, DNS is restricted to fairly simple geometries and low Reynolds number flows. Moreover, it is believed, see Mankbadi (1999), that large scales are more efficient than small ones in generating sound, which justifies the use of LES for sound predictions. To save computational time, a hybrid approach can be used in which the computational problem is divided into two parts. An LES can be used to obtain the unsteady non-linear near field, which in the jet noise case corresponds

to the hydrodynamic jet region. The acoustic field is then extended to far-field observer locations using, for example, the surface integration formulation by Kirchhoff (1883) as in e.g. Lyrantzis (1994) and Freund *et al.* (1996). Using a hybrid approach may even lead to noise predictions of higher quality than a direct approach since the amount of numerical dissipation to which the sound waves are exposed is decreased. Furthermore, the frequency cut-off is often higher in the integration surface regions than in the far field (Uzun, 2003; Uzun *et al.*, 2005). Another approach is to use a less computationally expensive Reynolds-Averaged Navier-Stokes (RANS) calculation to obtain a time-averaged flowfield. Information about length and time scales in the time-averaged flowfield can then be used to synthesize turbulence in the noise source regions. The acoustic field is obtained by solving a set of equations, e.g. the linearized Euler equations, with source terms obtained from the synthesized flowfield, see Billson (2004). This method is promising since it is possible to make simulations of high Reynolds number flows with reasonable computational efforts. Information on the acoustic far field can also be obtained by using RANS data as input for a statistical method based on an acoustic analogy as in e.g. Hunter & Thomas (2003), Farassat *et al.* (2004) and Self & Bassetti (2003) or by the use of RANS data in combination with semi-empirical relations as done by e.g. Tam & Auriault (1999), Tam & Pastouchenko (2002) and Tam & Ganesan (2004).

1.5 LES and DNS of Turbulent Jets

LES and DNS have been used for jet flow applications in a number of investigations. These primarily study jets at moderate Reynolds number due to the high computational costs of making simulations of a high Reynolds number jet. Many of these studies have been carried out to predict jet noise. However, as jet flows are free shear flows frequently occurring in both nature and industrial applications, it is interesting to study the jet in itself. Some studies are thus pure investigations of flow phenomena.

The feasibility of using LES for both the flowfield and the radiated sound from a high-subsonic 6.5×10^4 Reynolds number jet has been discussed by Bogey *et al.* (2000a, 2003, 2001). In the work presented in Bogey *et al.* (2000a, 2003), the acoustic field was obtained directly from the flow simulation. Noise-generation mechanisms were found to be relatively independent of the Reynolds number. In the work by Bogey *et al.* (2001) the acoustic analogy of Lighthill (1952) was used in combination with compressible LES to obtain the acoustic field. Bo-

gey & Bailly (2003, 2005a) investigated the effects of inflow conditions on the flowfield and the radiated sound of a high Reynolds number, $Re_D = 4.0 \times 10^5$, Mach 0.9 jet. Both the flow development and the emitted sound were shown to depend appreciably on inlet parameters. Bogey & Bailly (2004) investigated the effects of Mach and Reynolds number on subsonic jet noise. Bodony & Lele (2004) investigated the changes in the jet mean field and far-field sound when simulating jets with different temperatures and Mach numbers. Results were found to be in agreement with published data (Tanna, 1977). Freund (2001) used DNS to investigate sources of sound in a Mach 0.9 jet at a Reynolds number of $Re_D = 3.6 \times 10^3$. In this work the part of the Lighthill source that may radiate to the far field was isolated using Fourier methods. It was found that the peak of the radiating source coincides neither with the peak of the total source nor the peak of turbulence kinetic energy. Shur *et al.* (2003) made simulations of a cold Mach 0.9 jet at a Reynolds number of 1.0×10^4 . Radiated sound was successfully predicted using the surface integral formulation by Ffowcs Williams & Hawkings (1969). The simulation was made using only 5.0×10^5 cells. This work was done with the monotone-integrated LES (MILES) approach, where the subgrid-scale model is essentially replaced by the numerical dissipation of the numerical method. Zhao *et al.* (2001) made an LES of a Mach 0.9 jet at Reynolds number 3.6×10^3 and a jet at Mach 0.4 and a Reynolds number of 5.0×10^3 . In that study, radiated sound was obtained both directly from the LES and by using Kirchhoff surface integration. The effect on the radiated sound of the subgrid-scale model was investigated. It was found that using a mixed subgrid-scale model resulted in both higher turbulence levels and sound levels. Uzun (2003); Uzun *et al.* (2005) made LES of a high-subsonic (Mach 0.9) single-stream jet at a Reynolds number of 4.0×10^5 . Sound pressure levels obtained using surface integral methods, e.g. Kirchhoff and Ffowcs William & Hawkings, were compared to results obtained using a volume integral method (Lighthill's acoustic analogy). The results obtained using the surface approaches were found to be comparable with the results obtained using Lighthill integration.

1.6 Measurements Used for Validation

Measurements for the single-stream jet configuration were made by Jordan *et al.* at the MARTEL facility of CEAT (Centre d'Etudes Aérodynamiques et Thermiques), Poitiers. Two-component single-point and mono-component two-point measurements were made using Laser Doppler Velocimetry (LDV). The acoustic field was sampled using an arc of

microphones at 30 jet diameters and 50 jet diameters from the jet exit, respectively. These measurements were made within the framework of the JEAN project. For greater detail on the experimental set-up see Jordan *et al.* (2002*a,b*); Jordan & Gervais (2003). These measurements have also been reported in part by Power *et al.* (2004).

No experimental data for validation of the LES predictions for the coaxial jet configuration are available at this stage of the CoJeN project. At a later stage, single-point multi-component statistics and spatial correlations will be available for validation of the predicted flow data, and the predicted acoustic signature will be evaluated using far-field acoustic measurements.

Chapter 2

Large-Eddy Simulation

LARGE-EDDY SIMULATION has recently been established as a numerical approach for the simulation of turbulent flows in situations in which detailed information on turbulence characteristics and flow physics is desirable and DNS is far too expensive. LES enables accurate representation of large scales of the flow, i.e. as accurate as possible considering the errors introduced by the numerical method and the assumptions made. The effects on the large, resolved scales of those not resolved are modeled using a subgrid-scale model. By refining the mesh, LES approaches DNS, where all scales of the flow should be represented accurately. However, only resolving the larger scales significantly decreases the computational effort needed, making LES more feasible for non-academic flows. Furthermore, the larger eddies are directly affected by the boundary conditions, carry most of the Reynolds stresses and must be computed, whereas the small-scale turbulence is weaker, contributing less to the Reynolds stresses. Thus, it is less critical to represent these smaller scales accurately. The smaller scales are also more nearly isentropic and have nearly universal characteristics and are therefore more amenable to modeling (Wilcox, 1998).

2.1 Governing Equations

The compressible form of the continuity, momentum and energy equations in which the viscous stress and heat flux have been defined using Newton's viscosity law and Fourier's heat law, respectively, are often referred to as the Navier-Stokes equations:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u_i)}{\partial x_i} = 0 \quad (2.1)$$

$$\frac{\partial(\rho u_i)}{\partial t} + \frac{\partial(\rho u_i u_j)}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial \sigma_{ij}}{\partial x_j} \quad (2.2)$$

$$\frac{\partial(\rho e_0)}{\partial t} + \frac{\partial(\rho e_0 u_j)}{\partial x_j} = -\frac{\partial p u_j}{\partial x_j} + C_p \frac{\partial}{\partial x_j} \left(\left(\frac{\mu}{Pr} \right) \frac{\partial T}{\partial x_j} \right) + \frac{\partial}{\partial x_j} (u_i \sigma_{ij}) \quad (2.3)$$

σ_{ij} in Eqns. (2.2) and (2.3) is the viscous stress defined by

$$\sigma_{ij} = \mu \left(2S_{ij} - \frac{2}{3} S_{mm} \delta_{ij} \right) \quad (2.4)$$

where S_{ij} is the strain-rate tensor given by

$$S_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \quad (2.5)$$

Pr in Eqn. (2.3) denotes the Prandtl number.

In the present work the system of governing equations, Eqns. (2.1–2.3), is closed by making assumptions on the thermodynamics of the gas considered. It is assumed that the gas is a thermally perfect gas, i.e. it obeys the gas law

$$p = \rho RT \quad (2.6)$$

where R is the gas constant. Furthermore, the gas is assumed to be calorically perfect, which implies that internal energy and enthalpy are linear functions of temperature.

$$\begin{aligned} e &= C_v T \\ h &= C_p T \\ C_v &= C_p - R \end{aligned} \quad (2.7)$$

Here, C_p is the specific heat at constant pressure. Moreover, the viscosity, μ , is assumed to be constant.

2.2 Spatial Filtering

By low-pass filtering the governing equations in space, flow features that are smaller than an externally specified characteristic filter width are effectively removed from the solution. The basic idea is that structures larger than the filter width should be left unaffected by the filter operation and that the effect of scales smaller than the filter width are removed and must be modeled. In the present work the low-pass filtering procedure is considered to be coupled to the discretization in the finite-volume method solver, which means that it corresponds to a box filter and that the characteristic length scale is related to the local grid cell size.

2.2.1 Favre Filtering

A spatial Favre filter is used in this work, i.e. a mass weighted spatial filter. Favre filtering is a common filtering approach for compressible flows since it results in governing equations in a convenient form, i.e. a form very similar to that of the unfiltered equations. Moreover, using a filter approach in which fluctuations in density are considered would lead to more complicated subgrid-scale terms. For example, with such a filter, subgrid-scale terms would appear in the continuity equation, terms that would need modeling of their own (Guerts, 2004). Using Favre filtering, the flow properties are decomposed as follows

$$\Phi = \tilde{\Phi} + \Phi'' \quad (2.8)$$

where $\tilde{\Phi}$ is a Favre-filtered, resolved quantity and Φ'' is the unresolved part of Φ . The Favre-filtered part of Φ is obtained as follows

$$\tilde{\Phi} = \frac{\overline{\rho\Phi}}{\bar{\rho}} \quad (2.9)$$

in contrast to Favre time averaging

$$\tilde{\Phi} \neq \bar{\Phi} \quad (2.10)$$

and hence

$$\widetilde{\Phi''} \neq 0 \quad (2.11)$$

The Favre-filtered continuity, momentum and energy equations are obtained by applying the filtering operation, Eqn. (2.9), directly to the governing equations, Eqns. (2.1–2.3), which implies

$$\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial (\bar{\rho} \tilde{u}_i)}{\partial x_i} = 0 \quad (2.12)$$

$$\frac{\partial (\bar{\rho} \tilde{u}_i)}{\partial t} + \frac{\partial (\bar{\rho} \tilde{u}_i \tilde{u}_j)}{\partial x_j} = -\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial \bar{\sigma}_{ij}}{\partial x_j} + \frac{\partial \tau_{ij}}{\partial x_j} \quad (2.13)$$

$$\begin{aligned} \frac{\partial (\bar{\rho} \tilde{e}_0)}{\partial t} + \frac{\partial (\bar{\rho} \tilde{e}_0 \tilde{u}_j)}{\partial x_j} = & -\frac{\partial \bar{p} \tilde{u}_j}{\partial x_j} + \frac{\partial}{\partial x_j} \left(C_p \frac{\mu}{Pr} \frac{\partial \tilde{T}}{\partial x_j} + q_j^{SGS} \right) + \frac{\partial}{\partial x_j} (\overline{u_i \sigma_{ij}}) \\ & - \frac{1}{2} \frac{\partial}{\partial x_j} \bar{\rho} (\widetilde{u_i u_i u_j} - \widetilde{u_i} \widetilde{u_i} \tilde{u}_j) \end{aligned} \quad (2.14)$$

where $\bar{\sigma}_{ij}$ and τ_{ij} are the Favre-filtered viscous stress tensor and subgrid-scale viscous stress tensor, respectively. These are here defined as

$$\bar{\sigma}_{ij} = \mu \left(2\tilde{S}_{ij} - \frac{2}{3}\tilde{S}_{mm}\delta_{ij} \right) \quad (2.15)$$

and

$$\begin{aligned} \tau_{ij} &= -\bar{\rho} (\widetilde{u_i u_j} - \tilde{u}_i \tilde{u}_j) \\ &= -\bar{\rho} \left(\underbrace{(\widetilde{\tilde{u}_i \tilde{u}_j} - \tilde{u}_i \tilde{u}_j)}_I + \underbrace{(\widetilde{u_i'' \tilde{u}_j} + \widetilde{\tilde{u}_i u_j''})}_{II} + \underbrace{\widetilde{u_i'' u_j''}}_{III} \right) \end{aligned} \quad (2.16)$$

In Eqn. (2.15), $\tilde{S}_{ij}$ is the Favre-filtered rate of strain tensor given by

$$\tilde{S}_{ij} = \frac{1}{2} \left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i} \right) \quad (2.17)$$

As can be seen in Eqn. (2.16), the subgrid-scale stress tensor is a tensor built up of products of resolved and unresolved quantities. These terms appear when the spatial filter is applied to the convective term in the momentum equation. Terms I–III in Eqn. (2.16) are often referred to as Leonard stress, cross term and subgrid-scale Reynolds stress, respectively. When the convective terms in the energy equation are filtered, a term analogous to the subgrid-scale stress term appears. This term, denoted by q_j^{SGS} in Eqn. (2.14), is later referred to as a subgrid-scale heat flux term and is given by

$$\begin{aligned} q_j^{SGS} &= -C_p \bar{\rho} \left(\widetilde{T u_j} - \tilde{T} \tilde{u}_j \right) \\ &= -C_p \bar{\rho} \left(\widetilde{\tilde{T} \tilde{u}_j} - \tilde{T} \tilde{u}_j + \widetilde{T'' \tilde{u}_j} + \widetilde{\tilde{T} u_j''} + \widetilde{T'' u_j''} \right) \end{aligned} \quad (2.18)$$

Separating the velocity components into resolved and unresolved parts according to Eqn. (2.8), the last term in the Favre-filtered energy equation, Eqn (2.14), can be rewritten as follows:

$$\begin{aligned} \frac{1}{2} \frac{\partial}{\partial x_j} \bar{\rho} (\widetilde{u_i u_i u_j} - \widetilde{u_i u_i} \tilde{u}_j) &= \\ \frac{1}{2} \frac{\partial}{\partial x_j} \bar{\rho} \left(\underbrace{(\widetilde{\tilde{u}_i \tilde{u}_i \tilde{u}_j} - \widetilde{\tilde{u}_i \tilde{u}_i} \tilde{u}_j)}_I + \underbrace{(\widetilde{u_i'' u_i'' \tilde{u}_j} - \widetilde{u_i'' u_i''} \tilde{u}_j)}_{II} + \right. \\ \left. 2 \underbrace{(\widetilde{\tilde{u}_i u_i'' \tilde{u}_j} - \widetilde{\tilde{u}_i u_i''} \tilde{u}_j)}_{III} + \underbrace{\widetilde{u_i u_i u_j''}}_{IV} \right) \end{aligned} \quad (2.19)$$

The differences of triple correlations on the right hand side of Eqn. (2.19), terms I-III, can all be argued to be small and are thus neglected. Term IV is also assumed to be negligible and thus discarded, which means that all terms in Eqn (2.19) are neglected. Finally, term $(\overline{u_i \sigma_{ij}})$ in Eqn. (2.14) is replaced with $(\tilde{u}_i(\overline{\sigma_{ij}} + \tau_{ij}))$. With these assumptions made, there are two terms in the filtered equations that must be modeled, namely, τ_{ij} and q_j^{SGS} .

2.3 Subgrid-Scale Model

The subgrid-scale stress and heat flux defined by Eqns. (2.16) and (2.18), respectively, both contain products including unresolved quantities, that must be modeled. The main purpose of the subgrid-scale model is to provide dissipation of the resolved scales, i.e. correctly reproduce the transfer of energy from resolved to unresolved scales.

The simplest subgrid-scale models are so called eddy-viscosity models. These are based on the equilibrium assumption, i.e. small scales have shorter time scales than large, they adjust more rapidly to perturbations and they reach equilibrium almost instantaneously. Using these assumptions, the transport equation for the subgrid-scale kinetic energy reduces to

$$\varepsilon_\nu = -\tau_{ij} \overline{S_{ij}} \quad (2.20)$$

where ε_ν denotes viscous dissipation. The best known of these models is the Smagorinsky (1963) model, in which the anisotropic part of the subgrid-scale stress is represented as

$$\tau_{ij} - \frac{1}{3} \tau_{mm} \delta_{ij} = -2\nu_t \overline{S_{ij}} \quad (2.21)$$

The eddy-viscosity has the dimension $[m^2 s^{-1}]$ and can thus be represented by a length scale, ℓ , representing the subgrid scales and a velocity scale, $\mathcal{U} = \sqrt{\tau_{mm}/\rho}$. For an incompressible flow, assuming that the viscous dissipation and the eddy viscosity are proportional to the length scale and the velocity scale according to $\varepsilon_\nu \sim \mathcal{U}^3/\ell$ and $\nu_t \sim \ell \mathcal{U}$, respectively, Eqns. (2.20–2.21) gives $\mathcal{U} \sim \ell |\overline{S}|$. Finally, letting $\ell \sim \Delta$ gives the following expression for the eddy viscosity

$$\nu_t = (C_s \Delta)^2 |\overline{S}| \quad (2.22)$$

where C_s is the Smagorinsky constant.

The subgrid-scale model used in the present work is the Smagorinsky part of the model proposed by Erlebacher *et al.* (1992) for compressible flows. This model is based on Favre-filtered quantities, and the subgrid stress tensor is modeled as

$$\tau_{ij} = \mu_t \left(2\tilde{S}_{ij} - \frac{2}{3}\tilde{S}_{mm}\delta_{ij} \right) - \frac{2}{3}\bar{\rho}k^{SGS}\delta_{ij} \quad (2.23)$$

where k^{SGS} is the subgrid-scale kinetic energy given by

$$k^{SGS} = C_I \Delta^2 \tilde{S}_{mn} \tilde{S}_{mn} \quad (2.24)$$

and μ_t is the subgrid-scale kinematic viscosity defined as

$$\mu_t = C_R \bar{\rho} \Delta^2 \sqrt{\tilde{S}_{mn} \tilde{S}_{mn}} \quad (2.25)$$

The subgrid heat flux appearing in the Favre-filtered energy equation is modeled using a temperature gradient approach

$$q_j^{SGS} = C_p \frac{\mu_t}{Pr_t} \frac{\partial \tilde{T}}{\partial x_j} \quad (2.26)$$

The filter width, Δ , that appears in Eqns. (2.24) and (2.25) is represented in this study by the local grid cell width approximated by the cube root of the cell volume. In Paper III, the effects on shear layer flow structures and acoustic signature of changes in SGS dissipation were investigated by modifying the local SGS length scale. In the work reported in Paper IV it was found that, since the cells, inevitable, have a rather high aspect ratio in the near-nozzle regions, i.e. cells are significantly longer in the circumferential direction than in the radial direction, using $\Delta = (\Delta_1 \Delta_2 \Delta_3)^{1/3}$ as the local filter width introduced too great a subgrid-scale dissipation. Several possible representations of the filter width for cells with a high aspect ratio are described in Sagaut (2002). However, in paper IV, the shortest side of each cell was chosen as a rough estimate. In that study, decreasing the filter width to $\Delta = \min(\Delta_1, \Delta_2, \Delta_3)$ gave a flow development that seemed to be more physical.

Constants C_R and C_I that appear in Eqs. (2.24) and (2.25) are the Smagorinsky model constants, of which the latter is a compressibility correction constant. These are here given by

$$\begin{cases} C_R &= 0.012 \\ C_I &= 0.0066 \end{cases} \quad (2.27)$$

Chapter 3

Sound Propagation

SINCE the compressible Navier-Stokes equations include all information on the acoustic field, solving them directly gives information about the sound pressure levels in observer locations in the far-field regions. However, an approach of this kind is very expensive as concerns calculation effort. It is thus more common to use some kind of hybrid approach in which the problem is divided into two parts, i.e. first obtaining the instantaneous near field using a Navier-Stokes solver and then obtaining the far-field sound either by the use of an integral approach or by solving a set of equations, e.g. the Linear Euler Equations (LEE), with source terms obtained from the instantaneous flowfield, see e.g. Billson *et al.* (2005). Figure (3.1) gives a schematic overview of different CAA methods. The boxes defining the methodology used in the present work are filled with gray. Note that, in Figure (3.1), the concept acoustic analogy has been extended to include any method that uses different approaches to obtain flowfield information in the near field and acoustic information in the far field.

3.1 Lighthill's Acoustic Analogy

Sir James Lighthill's description of his acoustic analogy, i.e. the analogy between the full non-linear flow and the linear theory of acoustics, is often referred to as the start of aeroacoustics. Combining the time derivative of the continuity equation and the divergence of the momentum equation gives a scalar equation for an acoustic variable with a wave operator on the left-hand side (Lighthill, 1952).

$$\frac{\partial^2 \rho}{\partial t^2} = \frac{\partial^2}{\partial x_i \partial x_j} (\rho u_i u_j + p \delta_{ij} - \sigma_{ij}) \quad (3.1)$$

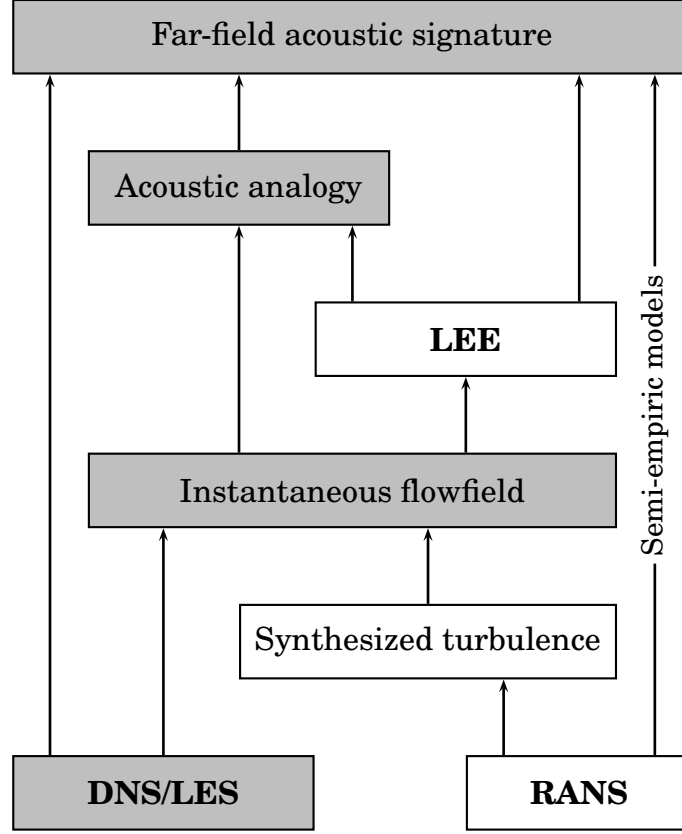


Figure 3.1: Computational aeroacoustic methods. Different methods for obtaining pressure signals in far-field observer locations are shown. The methodology used in this work is indicated by filled boxes.

By subtracting $c_\infty^2 \frac{\partial^2 \rho}{\partial x_i^2}$ on each side, Eqn. (3.1) can be rewritten as

$$\frac{\partial^2 \rho}{\partial t^2} - c_\infty^2 \frac{\partial^2 \rho}{\partial x_i^2} = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j} \quad (3.2)$$

which is the famous Lighthill's equation. In Eqn. (3.2), T_{ij} is the Lighthill stress tensor, defined by

$$T_{ij} = \rho u_i u_j + (p - \rho c_\infty^2) \delta_{ij} - \sigma_{ij} \quad (3.3)$$

An analytic solution to Eqn. (3.2) can be obtained by use of the free-space Green's function

$$G(\mathbf{y}, t) = \frac{1}{4\pi c_\infty^2 r} \delta(\tau_r) \quad (3.4)$$

which gives

$$\rho(\mathbf{y}, t) - \rho_\infty = \frac{1}{4\pi c_\infty^2} \int_V \frac{1}{r} \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}(\mathbf{x}, \tau_r) dV(\mathbf{x}) \quad (3.5)$$

where $\mathbf{y}$ is an observer location in the far field and $\mathbf{x}$ is the source location. τ_r denotes retarded time and is related to the observer evaluation time t as

$$\tau_r = t - \frac{r}{c_\infty} \quad (3.6)$$

where $r = |\mathbf{y} - \mathbf{x}|$ is the distance between a source location and an observer and c_∞ is the speed of sound in the far-field region. The left-hand side in Eqn. (3.5) corresponds to a far-field density fluctuation

$$\rho'(\mathbf{x}, t) = \rho(\mathbf{x}, t) - \rho_\infty \quad (3.7)$$

which, assuming that the acoustic compression-expansion processes in the flow are adiabatic, is related to a corresponding pressure fluctuation as

$$p'(\mathbf{x}, t) = \rho'(\mathbf{x}, t) c_\infty^2 \quad (3.8)$$

Using Eqn. (3.5) together with Eqn. (3.8), the sound pressure level in a far-field observer location can be estimated by integration over a volume containing all sound-generating sources. It is important to note that no interaction between the propagating sound wave and the flowfield in which it is propagating is explicitly considered when the integral form of the analogy is used, which for example means that no refraction or convection effects are taken into account. The equation produced by this pioneer work has later been modified in several ways to include, for example, the effects of the mean-flow acoustic interactions, flow inhomogeneities and solid surfaces. Among other, researchers such as Curle (1955); Lilly (1958); Ffowcs Williams (1963); Ffowcs Williams & Hawkings (1969); Ribner (1969) have made significant contributions to the theory proposed by Lighthill so as to make the use of the acoustic analogy applicable to a wider range of problems. Each of these theories differs in the definition of the noise source and the source definitions are all different from Lighthill's quadrupole source. Due to this non-uniqueness in noise source definition, it has recently been questioned by Tam (2001, 2002) whether the Lighthill quadrupole source term is a real noise sources or not. A thorough review of the development of the acoustic analogy is given in Crighton (1975).

3.2 Kirchhoff Surface Integration

Kirchhoff integration is a method for predicting the value of a property Φ governed by the wave equation at a point outside a surface enclosing all generating structures (Lyrintzis, 1994). The method was originally used in the theory of diffraction of light and in other problems of an electromagnetic nature (Kirchhoff, 1883) but has recently been extensively used for aeroacoustic applications. The integral relation is given by

$$\Phi(\mathbf{y}, t) = \frac{1}{4\pi} \int_S \left[\frac{\Phi}{r^2} \frac{\partial r}{\partial n} - \frac{1}{r} \frac{\partial \Phi}{\partial n} + \frac{1}{c_\infty r} \frac{\partial r}{\partial n} \frac{\partial \Phi}{\partial t} \right]_{\tau_r} dS(\mathbf{x}) \quad (3.9)$$

τ_r denotes that the expression within brackets is to be evaluated at a retarded time, i.e. emission time. c_∞ is again the speed of sound in the far-field region. The variable Φ that is to be evaluated in this case is the surface pressure. S denotes the surface enclosing all sound-generating structures and n denotes the direction normal to the surface. Surface S must be placed in a region where the flow is completely governed by a homogeneous linear wave equation with constant coefficients (Freund *et al.*, 1996).

The strength of the sources in the hydrodynamic jet region decays slowly downstream, which means that the downstream end of a closed surface will be located in regions of considerable hydrodynamic fluctuations. It is thus common practice to use Kirchhoff surfaces that are not closed in the upstream and downstream ends, see Figure (3.2). Freund *et al.* (1996) showed that the errors introduced by using such surfaces are small if the main portion of the sound sources are within the axial extent of the surface and if lines connecting observer locations with locations in the hydrodynamic region, representing the main sources of sound, intersect the surface. Rahier *et al.* (2003) found that the downstream closing surface makes only a minor contribution to the radiated sound. Greater detail on the Kirchhoff surface integration method is given in e.g. Freund *et al.* (1996) and Lyrintzis (1994).

All simulations described in this thesis have been done using jet-fitted Kirchhoff surfaces open in the downstream end. For each simulation the integration surface is defined by a set of grid nodes and expands in the radial direction as the mesh stretching increases. This prevents the surface from entering the hydrodynamic jet region when the jet grows in size downstream of the nozzle exit.

The Kirchhoff surface integral method is less computationally expensive and storage demanding than a method based on volume integrals, e.g. Lighthill's acoustic analogy, since it is only surface data that

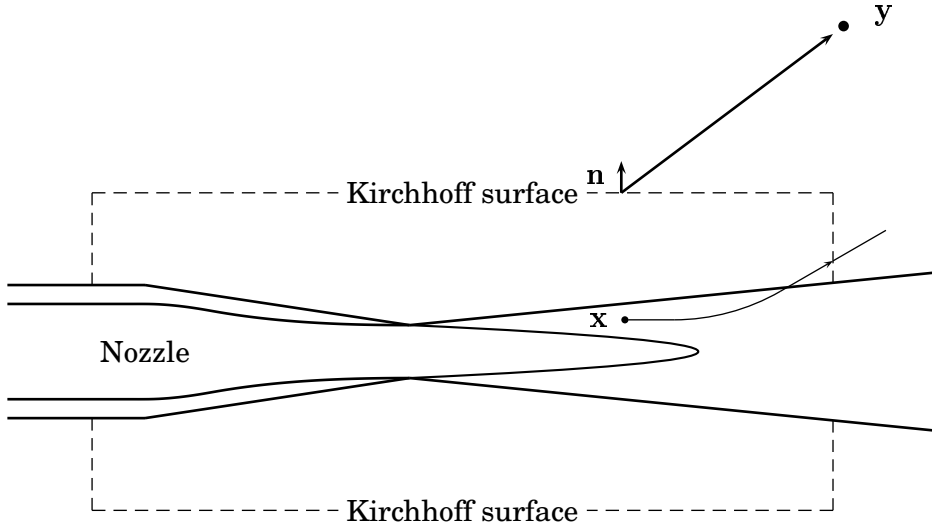


Figure 3.2: Sound generated in the shear-layer location, x , is propagated in the Navier-Stokes solver and extracted from the Kirchhoff surface to the far-field observer, y . The surface is open in the downstream end to prevent it from entering the hydrodynamic jet region.

must be stored and evaluated. Moreover, the integrand includes only first-order derivatives, while the integrand in Lighthill's acoustic analogy contains second-order derivatives in time or space depending on the formulation used. One of the main strengths of the Kirchhoff surface integral approach, contrary to many other methods, is that, as long as all sources of sound, flow inhomogeneities and objects that in some way affect the radiated sound are bounded by the surface, all these effects are taken into account. This means, for example, that no extra treatment is needed to be able to predict the effects of refraction and convection. This is one of the attractive features of this method. Moreover, both a strength and a weakness of the surface integral approach is the effect of numerical dissipation on the predicted sound. Acoustic waves approaching the integration surface will be low-pass filtered by the numerical method. A result of this artificial dissipation is that spurious high-frequency noise not supported by the mesh and scheme will be filtered out and thus not contribute to the predicted far-field sound. On the other hand, the numerical dissipation affects the ability to capture the high-frequency range of the acoustic pressure spectra, and thus the quality of the prediction depends strongly on the mesh resolution in the Kirchhoff surface region. The ideal situation would be to place the integration surface as far as possible from the sources to ensure that all non-linearities are enclosed by the integration sur-

face. However, the distance from the source to the surface is limited by the numerical dissipation introduced by the method. Using the surface integral method proposed by Ffowcs Williams & Hawkings (1969) on a porous surface formulation allows the surface to enter regions with weak hydrodynamic fluctuations, which means that the surface can be placed somewhat closer to the sources of sound than when the Kirchhoff formulation is used, see e.g. Shur *et al.* (2003), Rahier *et al.* (2003), Uzun (2003) and Uzun *et al.* (2005). If placed in the linear acoustic field, however, the Kirchhoff surface formulation and the formulation by Ffowcs William & Hawkings are identical. Moreover, Uzun (2003) and Uzun *et al.* (2005) found the results obtained using the two surface integration methods to be similar for integration surfaces placed rather close to the jet.

Chapter 4

Numerical Method

THE code used for the large-eddy simulations is one of the codes in the G3D family of finite-volume method, block-structured codes developed by Eriksson (1995). These codes solve compressible flow equations on conservative form on a general structured boundary-fitted, curve-linear non-orthogonal multi-block mesh. To enhance calculation performance and enable simulations with a large number of degrees of freedom, routines for parallel computations based on the MPI¹ libraries have been introduced in the code. Appendix C describes the MPI implementation in more detail.

This chapter gives an overview of the numerical scheme and boundary conditions used. Detailed descriptions of these subjects are given in Eriksson (1995) and Eriksson (2005).

For convenience, the Favre-filtered Navier-Stokes equations, Eqns. (2.12–2.14), are written in a more compact form:

$$\frac{\partial Q}{\partial t} + \frac{\partial \mathcal{F}_j}{\partial x_j} = 0 \quad (4.1)$$

where

$$Q = \begin{bmatrix} \bar{\rho} \\ \bar{\rho} \tilde{u}_i \\ \bar{\rho} \tilde{e}_0 \end{bmatrix} \quad (4.2)$$

and

$$\mathcal{F}_j = \begin{bmatrix} \bar{\rho} \tilde{u}_j \\ \bar{\rho} \tilde{u}_i \tilde{u}_j + \bar{p} \delta_{ij} - \bar{\sigma}_{ij} - \tau_{ij} \\ \bar{\rho} \tilde{e}_0 \tilde{u}_j + \bar{p} \tilde{u}_j - C_p \left(\left(\frac{\mu}{Pr} + \frac{\mu_t}{Pr_t} \right) \frac{\partial \tilde{T}}{\partial x_j} \right) - \tilde{u}_i (\bar{\sigma}_{ij} + \tau_{ij}) \end{bmatrix} \quad (4.3)$$

¹Message Passing Interface

The stress tensors, $\bar{\sigma}_{ij}$ and τ_{ij} , in Eqn. (4.3) are defined by Eqn. (2.15) and Eqn. (2.23), respectively. Eqn. (4.1) is discretized on a structured non-orthogonal boundary-fitted mesh. Integrating Eqn. (4.1) over an arbitrary volume Ω gives

$$\int_{\Omega} \frac{\partial Q}{\partial t} dV + \int_{\Omega} \frac{\partial \mathcal{F}_j}{\partial x_j} dV = 0 \quad (4.4)$$

Using the Gauss theorem to rewrite the second volume integral to a surface integral and introducing Q as the cell average of Q over Ω implies

$$\frac{\partial Q}{\partial t} V + \int_{\partial\Omega} \mathcal{F}_j \cdot d\mathcal{S}_j = 0 \quad (4.5)$$

In Eqn. (4.5), $\mathcal{S}_j = n_j \mathcal{S}$ is the face area normal vector, i.e. an area with a direction. The integral of the fluxes is approximated using the area of cell faces and the face average flux

$$\int_{\partial\Omega} \mathcal{F}_j \cdot d\mathcal{S}_j = \sum_{i=1}^{all\ faces} \mathcal{F}_j^i \cdot \mathcal{S}_j^i \quad (4.6)$$

Eqn. (4.4) can now be rewritten as

$$\frac{\partial Q}{\partial t} V + \sum_{i=1}^{all\ faces} \mathcal{F}_j^i \cdot \mathcal{S}_j^i = 0 \quad (4.7)$$

The total flux, $\mathcal{F}_j$, is divided into a convective part and a diffusive part in the following way:

$$\mathcal{F}_j = \begin{bmatrix} \bar{\rho} \tilde{u}_j \\ \bar{\rho} \tilde{u}_i \tilde{u}_j + \bar{p} \delta_{ij} \\ \bar{\rho} \tilde{e}_0 \tilde{u}_j + \bar{p} \tilde{u}_j \end{bmatrix} + \begin{bmatrix} 0 \\ -\bar{\sigma}_{ij} - \tau_{ij} \\ -C_p \left(\left(\frac{\mu}{Pr} + \frac{\mu_t}{Pr_t} \right) \frac{\partial \tilde{T}}{\partial x_j} \right) - \tilde{u}_i (\bar{\sigma}_{ij} + \tau_{ij}) \end{bmatrix} \quad (4.8)$$

where the first part is the convective, inviscid part and the second the diffusive, viscous part.

4.1 Spatial Discretization

4.1.1 Convective Fluxes

The convective flux across a given cell face is calculated with a user-defined upwind scheme based on the propagation direction of the characteristic variables at the cell face. This work has used a low-dissipation

third-order upwind scheme. The analysis of characteristic variables is made using the solution variables on primitive form q , which can be estimated from the variables on conservative form with second-order accuracy.

$$q = \begin{bmatrix} \bar{\rho} \\ \tilde{u}_i \\ \bar{p} \end{bmatrix} \quad (4.9)$$

Linearizing the governing equations on primitive form around a point located on the cell face and considering only plane waves parallel to the cell face gives a one-dimensional system of hyperbolic equations. This equation system can be decoupled by transformation to characteristic variables, see Appendix A. Estimates of the characteristic speeds are computed using face values of the state vector obtained as averages of the values in two adjacent cells according to

$$q^A = \frac{1}{2} (q_2 + q_3) \quad (4.10)$$

Superscript A in Eqn. (4.10) denotes average, and subscripts 2 and 3 denote the cells on each side of the face, see Figure (4.1).

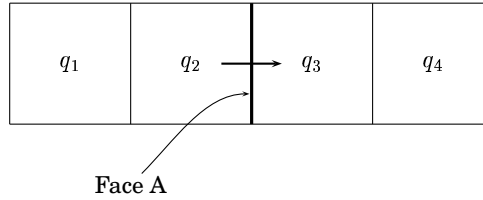


Figure 4.1: Cells used to obtain estimates of the inviscid fluxes across cell face A

Using the cell face values in Eqn. (4.10), the characteristic speeds are obtained as

$$\begin{aligned} \lambda_1 &= u_i^A \mathcal{S}_i \\ \lambda_2 &= \lambda_1 \\ \lambda_3 &= \lambda_1 \\ \lambda_4 &= \lambda_1 + c^A \sqrt{\mathcal{S}_i \mathcal{S}_i} \\ \lambda_5 &= \lambda_1 - c^A \sqrt{\mathcal{S}_i \mathcal{S}_i} \end{aligned} \quad (4.11)$$

The characteristic variables on the face are obtained using upwinded face values of the primitive variables. Two versions of q on the cell

face are calculated, one obtained by upwinding from the left and one obtained by upwinding from the right according to

$$\begin{aligned} q^L &= C_1 q_1 + C_2 q_2 + C_3 q_3 + C_4 q_4 \\ q^R &= C_4 q_1 + C_3 q_2 + C_2 q_3 + C_1 q_4 \end{aligned} \quad (4.12)$$

where superscripts L and R denote left and right, respectively. C_1, C_2, C_3 and C_4 are constants defining the upwinding scheme for a four-point stencil.

The sign of the characteristic speeds gives the direction in which local waves in the vicinity of the cell face are traveling in relation to the cell face normal vector. Each of the five characteristic variables are calculated using state vector values upwinded in the direction of the corresponding characteristic speed, i.e. either the left upwinded version or the right upwinded version of q is used. The characteristic variables are obtained as follows

$$\begin{aligned} w_1 &= \rho^{L/R} - \frac{p^{L/R}}{(c^A)^2} \\ w_2 &= \frac{\mathcal{S}_1 u_2^{L/R} - \mathcal{S}_2 u_1^{L/R}}{\sqrt{\mathcal{S}_1 \mathcal{S}_1 + \mathcal{S}_2 \mathcal{S}_2}} \\ w_3 &= \frac{1}{\sqrt{\mathcal{S}_i \mathcal{S}_i}} \left[\sqrt{\mathcal{S}_1 \mathcal{S}_1 + \mathcal{S}_2 \mathcal{S}_2} u_3^{L/R} - \mathcal{S}_3 \left(\frac{\mathcal{S}_1 u_1^{L/R} + \mathcal{S}_2 u_2^{L/R}}{\sqrt{\mathcal{S}_1 \mathcal{S}_1 + \mathcal{S}_2 \mathcal{S}_2}} \right) \right] \\ w_4 &= \frac{1}{2} \left[\frac{\rho^A}{c^A} \frac{\mathcal{S}_i u_i^{L/R}}{\sqrt{\mathcal{S}_j \mathcal{S}_j}} + \frac{p^{L/R}}{(c^A)^2} \right] \\ w_5 &= \frac{1}{2} \left[\frac{-\rho^A}{c^A} \frac{\mathcal{S}_i u_i^{L/R}}{\sqrt{\mathcal{S}_j \mathcal{S}_j}} + \frac{p^{L/R}}{(c^A)^2} \right] \end{aligned} \quad (4.13)$$

where again superscripts L and R denote left and right, respectively. The characteristic variables are transformed back to solution variables on primitive form using the following relations

$$\begin{aligned}
 \rho &= w_1 + w_4 + w_5 \\
 u_1 &= \frac{-\mathcal{S}_2 w_2}{\sqrt{\mathcal{S}_1 \mathcal{S}_1 + \mathcal{S}_2 \mathcal{S}_2}} - \frac{\mathcal{S}_1}{\sqrt{\mathcal{S}_j \mathcal{S}_j}} \left[\frac{\mathcal{S}_3 w_3}{\sqrt{\mathcal{S}_1 \mathcal{S}_1 + \mathcal{S}_2 \mathcal{S}_2}} - \frac{c^A}{\rho^A} (w_4 + w_5) \right] \\
 u_2 &= \frac{\mathcal{S}_2 w_2}{\sqrt{\mathcal{S}_1 \mathcal{S}_1 + \mathcal{S}_2 \mathcal{S}_2}} - \frac{\mathcal{S}_1}{\sqrt{\mathcal{S}_j \mathcal{S}_j}} \left[\frac{\mathcal{S}_3 w_3}{\sqrt{\mathcal{S}_1 \mathcal{S}_1 + \mathcal{S}_2 \mathcal{S}_2}} - \frac{c^A}{\rho^A} (w_4 + w_5) \right] \\
 u_3 &= \frac{1}{\sqrt{\mathcal{S}_j \mathcal{S}_j}} \left[\frac{w_3}{\sqrt{\mathcal{S}_1 \mathcal{S}_1 + \mathcal{S}_2 \mathcal{S}_2}} + \frac{\mathcal{S}_3 c^A}{\rho^A} (w_4 + w_5) \right] \\
 p &= (c^A)^2 (w_4 + w_5)
 \end{aligned} \tag{4.14}$$

The convective flux over the cell face can be estimated by inserting the face values obtained in Eqn. (4.14) into the inviscid part of Eqn. (4.8).

The convective scheme used in all the simulations reported in this thesis is a combination of centered and upwind biased components that have been used with good results for free shear flows by Mårtensson *et al.* (1991) and, more recently, for shock/shear-layer interaction by Wollblad *et al.* (2004) and Wollblad (2004). The coefficients of the low-dissipation upwind scheme used for estimating the convective flux over a cell face are derived using a third-order polynomial $Q(x)$ to represent the variation of the flow state in the direction normal to the face, see Figure (4.2).

$$Q(x) = A + Bx + Cx^2 + Dx^3 \tag{4.15}$$

The coefficients A , B , C and D are identified as functions of the cell-averaged states $Q_1 - Q_4$ by integration of Eqn. (4.15) over the corresponding cells. For more detail see Paper I.

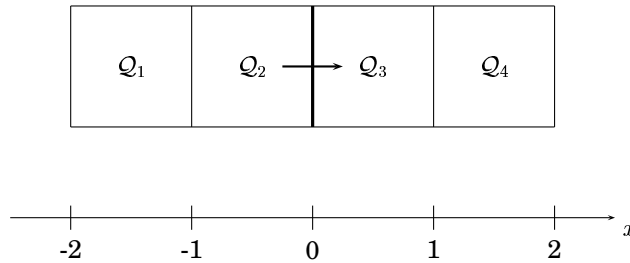


Figure 4.2: Face states, Q_0 , are estimated from the cell-averaged state in four neighboring cells.

The face state is evaluated using the interpolated value $Q(0)$, modified to include upwinding by adding the third derivative of $Q(x)$ according

to

$$\begin{aligned}\mathcal{Q}_0 &= Q(0) + \delta \frac{\partial^3 Q}{\partial x^3}(0) = A + 6\delta D = \\ &= C_1 \mathcal{Q}_1 + C_2 \mathcal{Q}_2 + C_3 \mathcal{Q}_3 + C_4 \mathcal{Q}_4\end{aligned}\tag{4.16}$$

where coefficient δ in front of the upwind term has been chosen by numerical experiments, Mårtensson *et al.* (1991), to be $1/96$ in order to introduce only a small amount of upwinding. The result is the low-dissipation third-order upwind scheme used in this work, i.e. a low-dissipation scheme compared to a standard third-order upwind scheme, which is obtained by using $\delta = 1/12$. With $\delta = 1/96$, the coefficients ($C_1 - C_4$) of the convective scheme are:

$$\begin{cases} C_1 = - \left(\frac{1}{12} + \delta \right) &= -9/96 \\ C_2 = \left(\frac{7}{12} + 3\delta \right) &= 59/96 \\ C_3 = \left(\frac{7}{12} - 3\delta \right) &= 53/96 \\ C_4 = - \left(\frac{1}{12} - \delta \right) &= -7/96 \end{cases}\tag{4.17}$$

Expressions for the dispersion and dissipation relations for the convective scheme can be derived by performing a semi-discrete stability analysis of a one-dimensional convection equation (Billson, 2004). One-dimensional convection can be described by the following relation:

$$\frac{\partial u}{\partial t} + c \frac{\partial u}{\partial x} = 0\tag{4.18}$$

where c is a constant coefficient. Assuming an initial solution to Eqn. (4.18) of a harmonic type, its exact solution can be written as

$$u(x, t) = u_0 \exp(st) \exp(-ikx)\tag{4.19}$$

where u_0 is a constant amplitude and k is the wave number. The time-dependent part of the solution is $\exp(st)$ and s describes its time dependence. Inserting Eqn. (4.19) in Eqn. (4.18) gives $s = ikc$, which implies that the exact solution to Eqn. (4.18) will have a periodic behavior in time without damping or amplification. Furthermore, the phase velocity will be the same for all wave numbers, namely, constant c .

An even-order central numerical approximation to a first-order derivative does not introduce any numerical dissipation in itself. Dissipation is thus often added in the range of wave numbers for which the dispersion errors are significant. Artificial numerical dissipation is introduced by adding an even-order derivative, which corresponds to adding $(-1)^{n+1} \delta (\partial^{2n} u / \partial x^{2n})$ on the right-hand side of Eqn. (4.18). Discretizing Eqn. (4.18) using a finite-volume method approach implies that first-order derivatives are estimated by differences in cell face

averages. Since the cell face-averaged properties in the solver used in the present work are obtained using a third-order upwind scheme where the upwinding is established by adding a third-order derivative, this corresponds to a fourth-order centered scheme with a fourth-order derivative added to introduce dissipation, in finite difference terms. Discretizing Eqn. (4.18) in space and introducing a finite difference approximation of the spatial derivative on an equidistant mesh using a five-point stencil, the time dependence s becomes

$$s = \frac{c}{\Delta x} \left[2i \sum_{l=1}^2 a_l^{FDM} \sin(lk\Delta x) - \delta \left(d_0^{FDM} + 2 \sum_{l=1}^2 d_l^{FDM} \cos(lk\Delta x) \right) \right] \quad (4.20)$$

where a_l^{FDM} corresponds to the finite difference coefficients defining the central scheme and d_l^{FDM} the finite difference coefficients defining the derivative added for upwinding. The amount of dissipation added is prescribed by constant δ . For more detail on the semi-discretization procedure see Billson (2004). Figure (4.3) shows the dispersion relation and dissipation relation of the scheme used in the present work. The dispersion relation is the imaginary part of Eqn. (4.20) and the dissipation relation is the real part. For comparison, the dispersion relation of the fourth-order dispersion relation preserving (DRP) scheme proposed by Tam & Webb (1993) is included in Figure (4.3(a)). The time stepping scheme will modify the dispersion and dissipation errors but, for the CFL² number chosen in the present study, $CFL \leq 0.5$, the effects are probably small.

As can be seen in Figure (4.3), the phase velocity is not the same for all wave numbers in the semi-discretized case. Higher-order schemes generally give a better representation of the phase velocity for a wider range of wave numbers. An expression for the number of grid points n needed per wavelength λ to give a correct representation of the phase velocity can be obtained by inserting $\lambda = n\Delta x$ in the definition of the wave number $k = 2\pi/\lambda$, which implies

$$n = \frac{2\pi}{k\Delta x} \quad (4.21)$$

Figure (4.3(a)) shows that, using the third-order low-dissipation scheme, the phase velocity starts to deviate from the analytical solution at $k\Delta x \simeq \pi/4$, which, using Eqn. (4.21), implies that 8 grid points are needed per wavelength. The DRP scheme proposed by Tam & Webb (1993) gives an accurate phase velocity representation for $k\Delta x \leq \pi/2$, which means that 4 grid points are needed per wavelength in this case.

²Courant-Friedrichs-Lewy

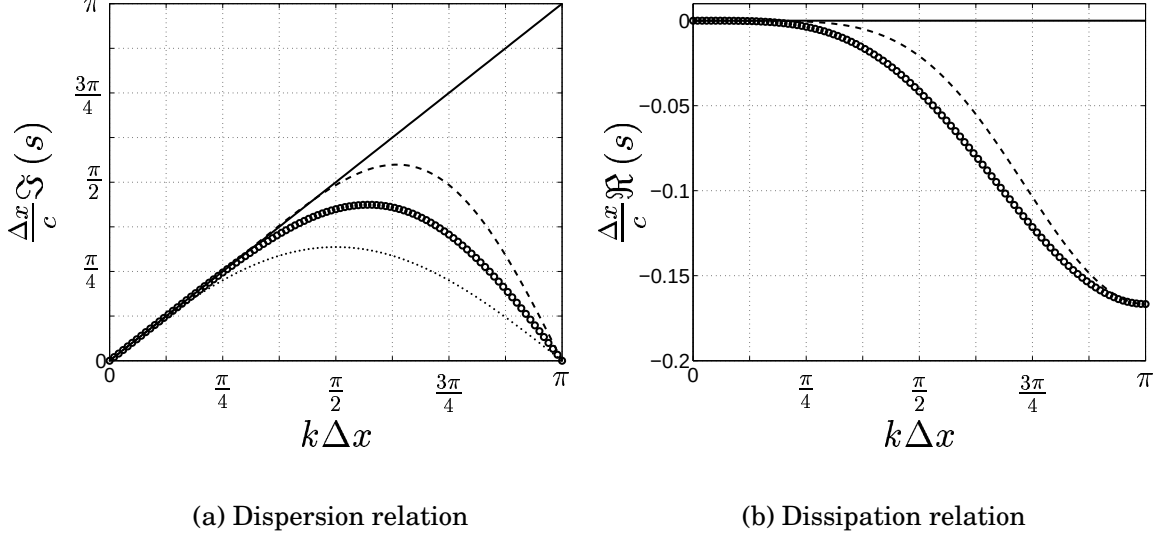


Figure 4.3: Circles denote the numerical scheme used in the present work and the solid lines denote the exact solution. In Figure (a), the dashed line corresponds to the fourth-order DRP scheme proposed by Tam & Webb (1993) and the dotted line is the dispersion relation for a second-order centered scheme. In Figure (b), the circles denote the dissipation relation for a fourth-order derivative with amplifying factor $\delta = 1/96$ and the dotted line represents the same fourth-order derivative with $\delta = 1/12$, i.e. the standard third-order upwind scheme.

4.1.2 Diffusive Fluxes

Evaluation of the viscous fluxes requires estimates of spatial derivatives of the primitive variables on the cell faces. Derivatives of the primitive variables in computational space are obtained using a centered difference approach. These derivatives are then translated to derivatives in physical space using a chain rule relation given by

$$\frac{\partial q}{\partial x_i} = \frac{\partial \xi_j}{\partial x_i} \frac{\partial q}{\partial \xi_j} \quad (4.22)$$

where ξ_i denotes the coordinates in computational space. This procedure is described in more detail in Appendix B. The derivatives in computational space are calculated as follows

$$\begin{aligned}
 \frac{\partial q}{\partial \xi_1} &= q_{i+1,j,k} - q_{i,j,k} \\
 \frac{\partial q}{\partial \xi_2} &= \frac{1}{4} (q_{i,j+1,k} - q_{i,j-1,k}) + \frac{1}{4} (q_{i+1,j+1,k} - q_{i+1,j-1,k}) \\
 \frac{\partial q}{\partial \xi_3} &= \frac{1}{4} (q_{i,j,k+1} - q_{i,j,k-1}) + \frac{1}{4} (q_{i+1,j,k+1} - q_{i+1,j,k-1})
 \end{aligned}
 \tag{4.23}$$

where again ξ_i represents the coordinates in computational space and subscripts i, j and k denote the cell index in three directions, see Figure (4.4). Using these estimates of spatial derivatives, the viscous flux contribution can be calculated using the diffusive part of Eqn. (4.8).

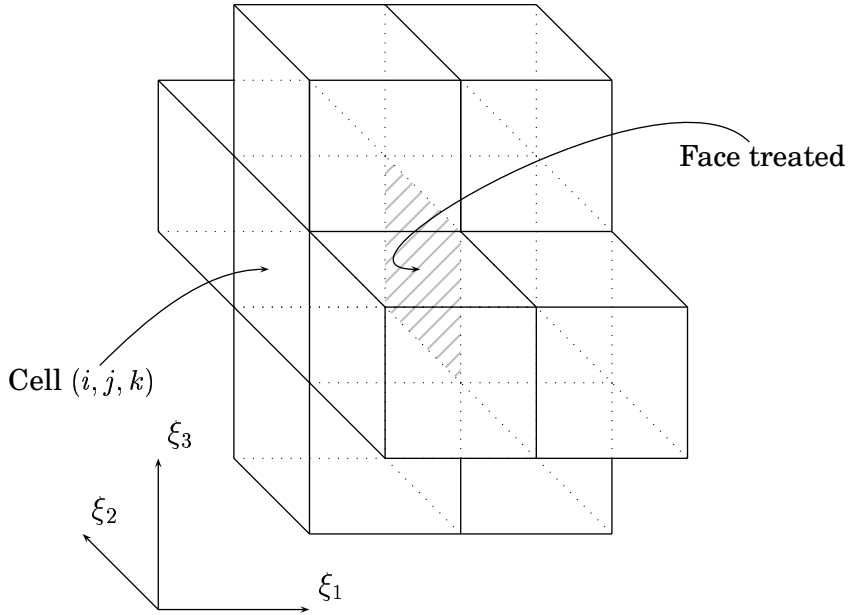


Figure 4.4: Spatial derivatives on a cell face in computational space are estimated using ten neighboring cells.

4.2 Time Stepping

Time stepping is performed by a three-stage second-order low-storage Runge-Kutta method. When all convective flux and diffusive flux contributions have been calculated as described in the previous sections,

the temporal derivative of the flow variables in a certain cell can be estimated as the net flux, i.e.

$$\frac{\partial \mathcal{Q}^n}{\partial t} = \mathcal{F}^n \quad (4.24)$$

The time stepping is done in three stages by calculating the state vector, $\mathcal{Q}$, for two intermediate sub-time steps. The time stepping algorithm is as follows

$$\begin{aligned} \mathcal{Q}^* &= \mathcal{Q}^n + \Delta t \mathcal{F}^n \\ \mathcal{Q}^{**} &= \frac{1}{2} [\mathcal{Q}^n + \mathcal{Q}^* + \Delta t \mathcal{F}^*] \\ \mathcal{Q}^{n+1} &= \frac{1}{2} [\mathcal{Q}^n + \mathcal{Q}^* + \Delta t \mathcal{F}^{**}] \end{aligned} \quad (4.25)$$

where superscript n denotes the previous time step and $n + 1$ the next time step. Superscripts $*$ and $**$ denote sub-time steps. Δt is the solver time step, i.e. the time between stage n and $n + 1$.

4.3 Boundary Conditions

This section will give a short description of the boundary conditions used in the work reported in Papers I–IV. Total pressure and total enthalpy are specified at the inlet of the nozzles, the nozzle plenum. All free boundaries, i.e. upstream and entrainment boundaries, are defined using absorbing boundary conditions based on characteristic variables. Static pressure is specified at the domain outlets. Figures (4.5) and (4.6) show the main boundary condition locations for the single-stream jet simulations (Papers I–III) and for the dual-stream simulations (Paper IV), respectively.

4.3.1 Method of Characteristics

A well-posed, non-reflecting, zero-order boundary condition may be specified by analyzing the behavior of local planar waves in the vicinity of the boundary. Close to the boundary, all planar waves can be described by the five characteristic variables, see Appendix A. These five characteristic variables are interpreted physically as one entropy wave, two vorticity waves and two acoustic waves. By obtaining the characteristic speed corresponding to each of the characteristic variables, a well-posed inflow/outflow condition can be specified. The sign of a characteristic speed indicates the direction in which information is transported

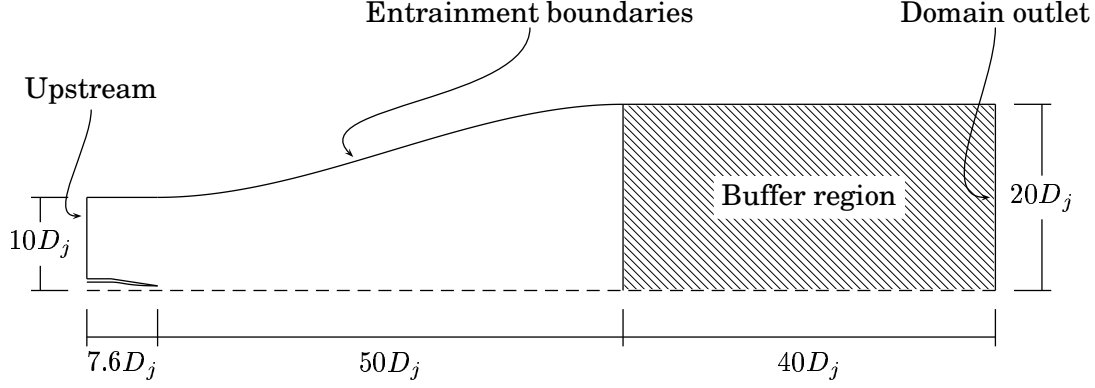


Figure 4.5: Computational set-up – single-stream jet. D_j is the nozzle outlet diameter ($D_j = 0.05[m]$).

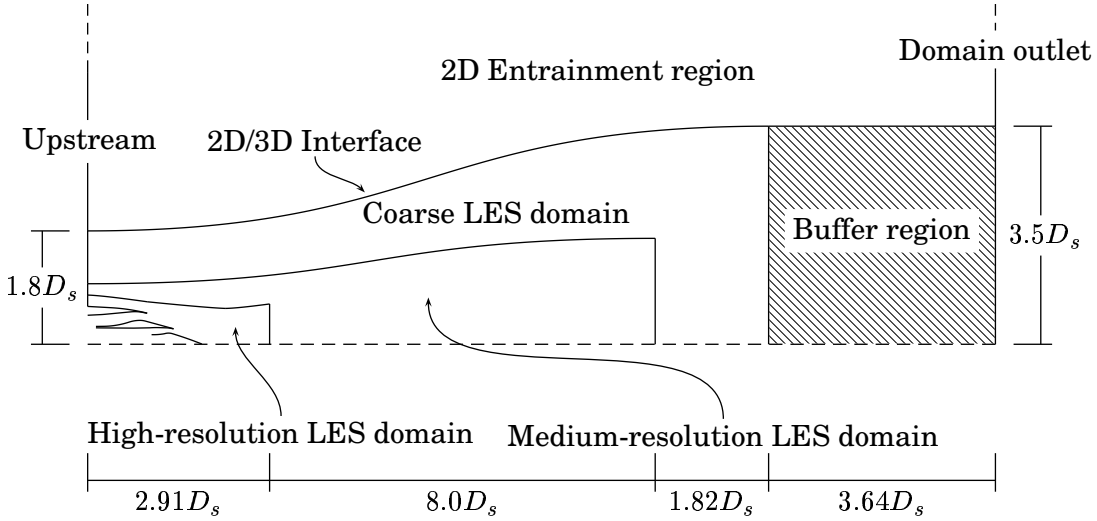


Figure 4.6: Computational set-up – dual-stream jet. D_s is the secondary stream nozzle outlet diameter ($D_s = 0.275[m]$).

over the boundary. Negative speed means that information is traveling into the domain and information about the exterior state has to be specified for the corresponding characteristic variable. A positive sign for the characteristic speed indicates that information is traveling through the boundary and out from the domain, which means that exterior information can not be specified but must be extrapolated from

the interior.

For outgoing waves to be totally absorbed, the transport direction must be aligned with the direction of analysis, i.e. usually aligned with the boundary normal vector. A wave reaching the boundary with an incident angle of transport will be partly reflected at the boundary, see Figure (4.7).

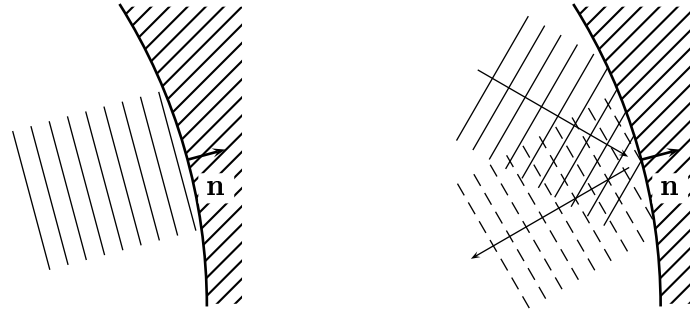


Figure 4.7: A plane wave reaching the boundary with the transport direction aligned with the surface normal is perfectly absorbed (left), whereas a wave not aligned will be partly reflected (right).

4.3.2 Special Treatment of Outlet Boundaries

Defining proper boundary conditions is an issue of great importance, especially in aeroacoustic applications, since acoustic pressure fluctuations are small and spurious waves generated at the boundaries might contaminate the acoustic field. Definitions of boundary conditions for free shear flows are particularly difficult, since there are by definition no bounding surfaces. The difficulties in defining boundary conditions for free shear flows have been discussed in several texts (e.g. Colonius *et al.*, 1993; Bogey *et al.*, 2000b; Mankbadi *et al.*, 2000; Rembold *et al.*, 2002).

When a jet flow is simulated, boundary conditions that approximate the jet behavior at infinity are to be specified at a finite distance downstream of the nozzle exit. Energetic vorticity and entropy waves traveling out of the domain reaching the outlet boundary will, if not damped out, generate strong acoustic waves traveling back into the domain. These acoustic reflections can be diminished by in some way decreasing the amplitude of fluctuations of vorticity and entropy waves approaching the boundary. Therefore, an outlet zone with extra damping is often added to the computational domain in the simulation of free shear flows. Inward-traveling acoustic waves may still be generated

but will be weaker than in the case of a boundary for which no special treatment of the boundary region is utilized. Moreover, these weaker acoustic waves are further damped as they travel through the outlet region on their way back into the calculation domain; hopefully, the waves are so weak when they reach the physical part of the computational domain that they do not affect the acoustic field. The functionality of the damping zones used in the present work (Figures (4.5)–(4.6)) is described in more detail in Paper I.

4.3.3 Entrainment Boundaries

At the entrainment boundaries an inflow corresponding to the fluid entrained by the jet has to be specified. The effect of the entrained mass flow not being correct is that a deficit of mass is compensated for by an inflow of fluid from the domain outlet. This back-flow results in a recirculation zone surrounding the jet and prevents it from spreading. For the single-stream jet, approximate values of the velocities specified at the entrainment boundaries were obtained from 2D axisymmetric RANS calculations (Eriksson, 2002) made for the same nozzle geometry and flow conditions. These RANS calculations were made using a significantly larger calculation domain than in the LES and therefore give reliable information on the entrainment flow at the boundary of the LES domain, see Figure (4.8). Furthermore, the RANS calculations have been extensively validated against experimental data (Jordan *et al.*, 2002a; Power *et al.*, 2004). Two RANS flowfields were used, one for the unheated jet and one for the heated.

It was assumed for in dual-stream simulations that the radial extent of the computational domain increases downstream such that the flow in the domain boundary region can be assumed to be irrotational and axisymmetric. Hence, the flow outside the three-dimensional computational domain can be represented by a less expensive two-dimensional axisymmetric calculation, see Figure (4.6). The domain for this 2D calculation extends 4.5 meters radially from the jet centerline. To minimize boundary reflections, the 2D/3D interface acts as an absorbing boundary condition on both sides. The flow properties of the boundary conditions for the 3D domain are specified using flow information from the 2D domain, and the boundary condition for the 2D domain is defined using circumferentially-averaged flow information from the 3D domain, see Figure (4.9). The 2D domain has an extent in the tangential direction corresponding to the width of the scheme used for the convective fluxes, i.e. four cells, which simplifies the implementation of the boundary treatment in the code. Azimuthal averages are obtained

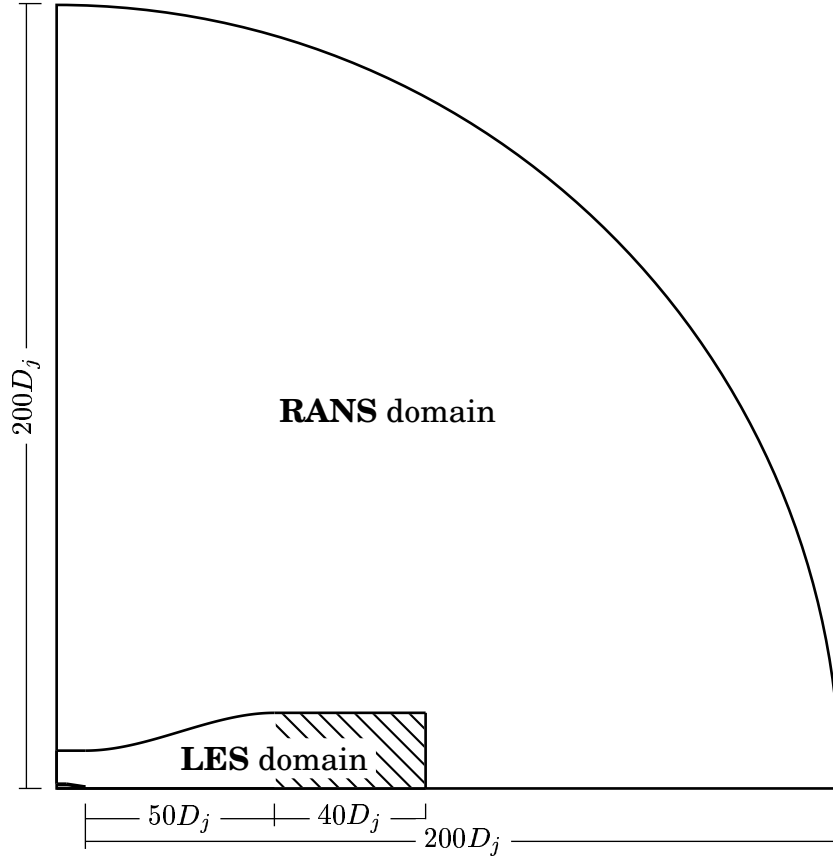


Figure 4.8: Computational domains for the LES and the RANS calculations of the single-stream jet

for the four cell rows adjacent to the interface highlighted in Figure (4.9). For the 2D domain, rotational periodic interfaces are used for the block faces in the tangential direction. This entrainment boundary approach enables the use of a rather narrow 3D domain. Furthermore, the outer boundaries of the 2D domain are located far from the jet axis, which simplifies the definition of these boundary conditions. The interface treatment is based on the mixing-plane technique reported by Stridh (2003).

4.4 Evaluation at Retarded Time

The concept of retarded time evaluation is shown schematically in Figure (4.10). The upper row of cells in the figure represents the ob-

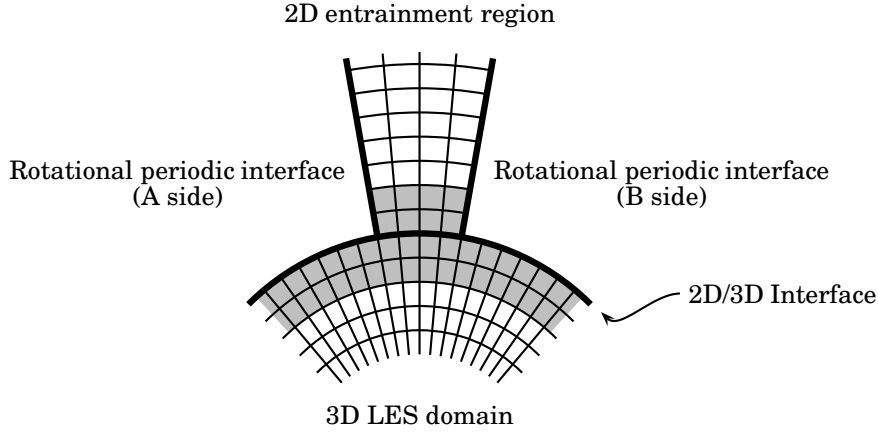


Figure 4.9: Interface between the 3D LES domain and the "2D" entrainment region in the dual-stream jet computational set-up.

server pressure signal and the lower rows represent the history of sound sources for four surface elements. Each cell in the four lower rows denotes an instant in time and represents a mean of a number of solver time steps. The lowest of these four sources is located farthest away from the observer. This is the source location that will define the starting point of the observer pressure signal since the first observer pressure sample that gets a contribution from the source location farthest away from the observer will be the first complete sample. In the same way, the source location closest to the observer defines the temporal extent of the complete pressure signal since this source will give the last contribution to the signal. Cells marked with a black point do not contribute to the observer pressure signal and the cells filled with gray indicate the time steps for which data must be stored in the solver. Five time steps are needed since fourth-order accurate temporal derivatives are used to evaluate the expression given in Eq. 3.9. The observer pressure signal is composed of a number of pressure samples separated in time by a predefined time increment. The arrival time of sound waves is defined by the generation time, the speed of sound and the distance to the observer. It is most unlikely that the arrival time will match the predefined discrete instants representing the pressure signal. Thus each surface element contributes to the pressure signal at two discrete times in the observer pressure signal, i.e. the two instants in time closest to the arrival time. The contribution to each of these is obtained by interpolation. The retarded time integration has been implemented such that only the observer pressure signals are stored, which is much less memory consuming than it would be to store the surface data and calculate the pressure signal afterward.

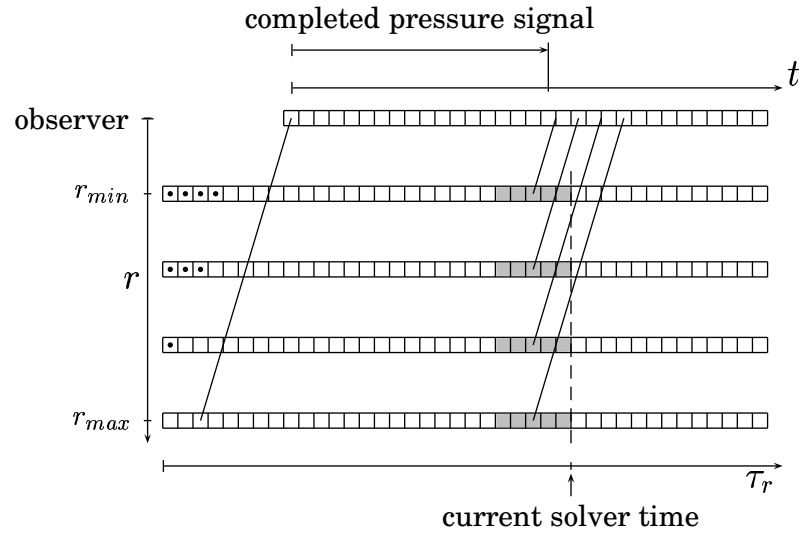


Figure 4.10: The upper row of cells represents the observer pressure signal and the lower ones the time series of sound source parameters for four surface or volume elements. τ_r denotes retarded time.

Chapter 5

Summary of Papers

THIS chapter gives a short summary of the work done and the results reported in the four papers on which this thesis is based.

5.1 Paper I

5.1.1 Motivation and Background

Paper I gives a presentation of predicted near-field flow data and far-field acoustics of a single-stream isothermal Mach 0.75 jet. The main reason for choosing to simulate a jet at that Mach number was that measurements were available for both isothermal and heated conditions, which gave the possibility to, in the continuation of the work reported in Paper I, investigate whether or not the method copes with the increased complexity associated with hot jets.

5.1.2 Work and Results

LES were made of a Mach 0.75 nozzle/jet configuration. The Reynolds number based on the nozzle-exit diameter and the jet velocity at the nozzle-exit plane, Re_D , was 5.0×10^4 . The Reynolds number in the measurements (Jordan *et al.*, 2002a; Jordan & Gervais, 2003; Jordan *et al.*, 2002b; Power *et al.*, 2004) used for comparison and validation of the simulations was approximately one million. Such a high Reynolds number probably means that the scales that must be resolved are too small to be captured with a reasonable computational effort. Thus the Reynolds number in our LES was decreased on the assumption that the flow is only weakly Reynolds number dependent.

The computational domain consists of a boundary-fitted block structured mesh with 50 blocks and a total of about 3.0×10^6 cells. The

grid cells are concentrated to the shear-layer area. To establish mesh homogeneity, a combination of polar and Cartesian mesh blocks was used. It should be noted that no inlet forcing was utilized and that the nozzle geometry was included in the calculation domain. Approximate values of the velocities specified at the entrainment boundaries were obtained from RANS calculations, which enabled the use of a rather narrow computational domain for the LES.

For validation of the LES results, the predicted time-averaged flow quantities and two-point space-time correlations were compared to corresponding measured quantities. Although some deviations occurred, the results were in generally good agreement with experiments. The initial jet spreading and the potential core length were not predicted correctly, however. An overpredicted $u'v'$ correlation close to the nozzle exit indicates that the degree of mixing is too high in this region, which makes the transition process too rapid and hence the length of the potential region becomes too short. The discussion in this paper focuses to a large extent on possible explanations for the fact that the predicted potential core is shorter than the measured.

Predicted sound pressure levels were in excellent agreement with measured levels (Jordan *et al.*, 2002b). For most observer locations the predicted sound pressure levels were within a deviation of 1.0 dB from the measured levels. However, the mesh used failed to capture waves of Strouhal numbers higher than $St \simeq 1.2$. This results in a rapid fall-off of the spectrum amplitudes for higher Strouhal numbers. However, the lower part of the spectra was accurately captured. The main conclusion regarding the predicted acoustic signature was that since the predicted sound pressure levels were in good agreement with the measured levels, capturing the lower part of the spectra seems to be sufficient to represent the major portion of the radiated sound.

5.1.3 Comments

At this stage, the main reason for the overpredicted shear layer mixing and thus shorter potential core was believed to be a lack of subgrid-scale dissipation. The study reported in Paper III however gives indications that this might not be the case.

Although the predictions of both near-field statistics and far-field acoustic signature are in generally good agreement with measured data, there are a number of parameters for which the predictions are sensitive. For example, using higher-order schemes or an increase of the mesh resolution would probably improve both the predicted flow statistics and the predicted acoustic signature.

The fact that the predictions have been obtained for a lower Reynolds number than the measured data changes the range of scales present in the jet. This will primarily affect the frequency content of the radiated sound. The inflow conditions affect the initial mixing, and thus the initial shear layer growth and the potential core length are affected. Moreover, since the current effort deals with a subsonic jet, most of the radiated sound is generated in the potential core region, which is highly dependent on the nature of the incoming disturbances. Both the Reynolds number effects and the effects of the inflow conditions on the radiated sound are probably noticeable mostly in the high-frequency range, however, which is significantly filtered by the numerical method.

Another uncertainty is related to the location of the Kirchhoff surface. The predicted sound pressure levels are sensitive to the location of the integration surface. The surface must be placed far enough from the jet to avoid hydrodynamic fluctuations in the surface region but close enough to avoid numerical dissipation of the radiated sound. To accomplish a good compromise, a jet-fitted integration surface was used. Nonetheless, the surface location should be considered a factor of uncertainty.

5.2 Paper II

5.2.1 Motivation and Background

In Paper II the predicted near-nozzle flowfield and radiated sound of the isothermal Mach 0.75 jet described in Paper I were compared to predictions obtained for a heated jet at the same Mach number. The jet-to-ambient temperature ratio of the heated jet was equal to 2.0. The predictions were compared with experimental data and the effects of increasing the temperature of the jet stream were investigated.

5.2.2 Work and Results

The computational set-up in this study was the same as was used for the study reported in Paper I and will not be discussed in more detail in this section.

The maximum levels of resolved turbulence intensities were in good agreement with measured data for both jets. Furthermore, the turbulence anisotropy was correctly predicted. For the heated jet, as for the isothermal jet, the initial jet spreading and the potential core length were not correctly predicted. In general the predicted flowfield is, however, considered to be in good agreement with measurements.

Two-point space-time correlations were obtained in a number of locations in the shear layer along the nozzle lip-line. Correlation curves obtained at an axial location corresponding to the end of the potential core were in good agreement with experiments by Jordan & Gervais (2003). Two-point space-time correlations in the shear layer were used to obtain estimates of integral length scales, integral time scales and eddy convection velocities. There was satisfying agreement with corresponding quantities obtained from the measured data.

Only small differences could be identified in correlations and integral scales for the two jets. Predicted sound pressure levels were in excellent agreement for both jets with the levels measured by Jordan *et al.* (2002b).

5.2.3 Comments

It was found in the work reported in this paper that the changes in the acoustic signature caused by the higher jet temperature in the latter of the two simulations are almost insignificant. The change in far-field sound pressure levels was probably too small to be correctly captured and thus no conclusions on the heating effects on the radiated sound were drawn in this study. The classic work of Tanna (1977) shows that Mach 0.75 falls almost perfectly in the range of Mach numbers for which the effect of heating the jet stream on the acoustic signature changes from increased SPL to decreased SPL, which means that the changes in radiated sound are very small, see Figure (5.1). This should probably have been considered when the test matrix was defined. For example, Mach 0.9 would have been a better choice from several aspects. First of all, as can be seen in Figure (5.1), the heating effect is more pronounced at this Mach number. Moreover, Mach 0.9 is a Mach number for which both predictions and measurements of jet flowfield and radiated sound have been reported extensively in the literature and thus more data is available for validation.

5.3 Paper III

5.3.1 Motivation and Background

The effects of modifying the subgrid-scale model and the inlet conditions on the predicted near-nozzle flow structures and the radiated sound of the single-stream jet presented in Paper I are investigated.

From the studies reported in Papers I and II it seems that, for a model nozzle, capturing the initial turbulent shear flow might not be

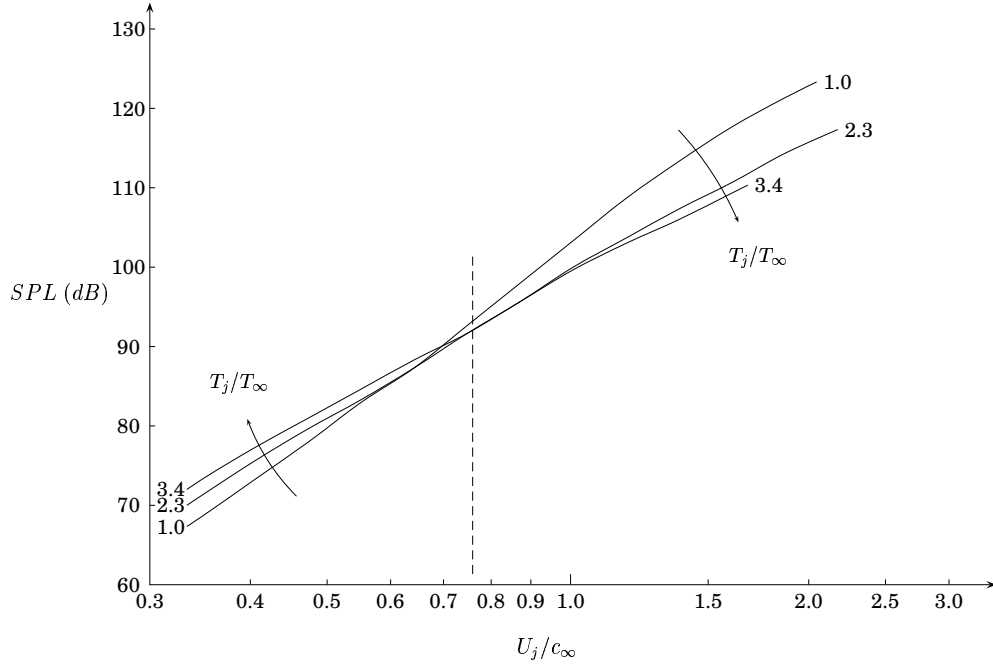


Figure 5.1: Sound pressure levels as a function of jet Mach number for different jet to ambient temperature ratios at $\theta = 90^\circ$ (θ being the angle from the x -axis). Measurements by Tanna (1977).

of very great importance for an accurate prediction of radiated sound since the major portion of the sound generated by the fine-scale shear-layer turbulence will appear in the high-frequency range. When dealing with real engine geometries, however, it becomes quite important due to the shift in frequencies related to the change in geometry and thus range of scales. Moreover, for prediction techniques to be useful to industry, the methods must cope with realistic jet flows, such as coaxial jet configurations with high velocities, significant velocity and temperature gradients, and arbitrary nozzle geometries. Hence, being able to capture the initial flow physics becomes even more important. Since it was found in Papers I and II that the predicted shear layer development differed somewhat from measurements obtained for the same nozzle configuration and flow properties, the study reported in this paper was performed.

5.3.2 Work and Results

Short descriptions of the simulations for which results are reported in Paper III are listed below.

- Simulation 1** Reference simulation (the isothermal Mach 0.75 jet presented in Papers I and II)
- Simulation 2** The simulated Reynolds number in this simulation corresponds to the Reynolds number for which the measurements were obtained.
- Simulation 3** Synthesized inlet turbulence was added at the nozzle inlet.
- Simulation 4** A Leonard flux correction term was implemented so as to give a better representation of the subgrid-scale stresses. The flux correction term approximates the effect of the Leonard stresses, i.e. term I in Eqn. (2.16).
- Simulation 5** The effects of increased filter width in the subgrid-scale model was investigated.
- Simulation 6** The effects of decreased filter width in the subgrid-scale model was investigated.
- Simulation 7** The total pressured specified at the nozzle inlet was given as a radial profile. In the other simulations, a constant total pressure was specified.

Of the simulations reported in this paper only the one with increased filter width gave significant changes in the near-nozzle flow statistics. The increased subgrid-scale dissipation results in a lower degree of mixing close to the nozzle than for the other simulations presented. This in turn leads to a more violent mixing process once initiated, which can be seen in the higher levels of resolved turbulence intensities and an overpredicted $u'v'$ correlation for this jet. As a consequence, the predicted sound pressure levels are significantly increased. The potential core is somewhat shorter in this simulation than in the others, which is consistent with the higher degree of mixing. Moreover, the rate of decay of the centerline axial velocity is slightly increased. The most striking result in this study is that none of the changes from the reference simulation gave a potential core length in better agreement with the measured data.

The main observation regarding the ability to predict the radiated sound was that introducing flux correction terms that approximate the effects of the Leonard stresses appearing in the spatially filtered governing equations gave a better representation of the high-frequency content of the radiated sound. Observer locations situated on the forward arc are affected to a higher degree than the rear arc observers by

high-frequency noise generated in the shear layer region. Thus the improved representation of the higher frequency content results in sound pressure levels that are better aligned with the measured levels for the upstream observers.

5.3.3 Comments

It is interesting to note that, although some changes could be identified in the snapshots of near-nozzle flow structures, the changes in near-field statistics were insignificant. It was discussed by Sagaut (2002) that, although LES might be able to capture the physics of shear layer flows, quantities such as the potential core length in the jet case are almost never captured. In summary, it seems that, although some changes to the shear layer flow can be observed for the simulations made in this work, the flow statistics in the near-nozzle region is rather insensitive to changes in inflow parameters and subgrid-scale quantities.

As mentioned previously, implementation of the Leonard flux correction terms gave a better representation of the sound radiated to the forward arc. It should be noted, however, that this slightly improved prediction of sound radiated to the forward arc observers comes with a penalty in terms of calculation cost. The iteration wall time when the Leonard flux correction was implemented in the solver was roughly 4/3 times the iteration time for the other simulations. Hence, it might be valuable to consider a mesh refinement with a similar increase in calculation time.

5.4 Paper IV

5.4.1 Motivation and Background

Paper IV gives a presentation of predicted near-field flow and acoustic signature of a dual-stream nozzle/jet configuration. A high-subsonic coaxial jet with heated primary stream is studied. The static temperature in the primary stream (core flow) is roughly three times that of the secondary stream (bypass flow), leading to large temperature gradients in the shear layer between the two streams. The jet Mach number, $M_j = U_j/c_j$, is approximately 0.9 for both streams. The jet is expanded into stagnant surroundings and the temperature of the secondary stream is equal to the ambient temperature.

The main objective of the study reported in Paper IV was to investigate the feasibility of using LES for a coaxial nozzle/jet configuration

of high complexity in terms of nozzle geometry, jet velocity and velocity and temperature gradients.

Figure (5.2) gives a schematic illustration of the development of a dual-stream jet.

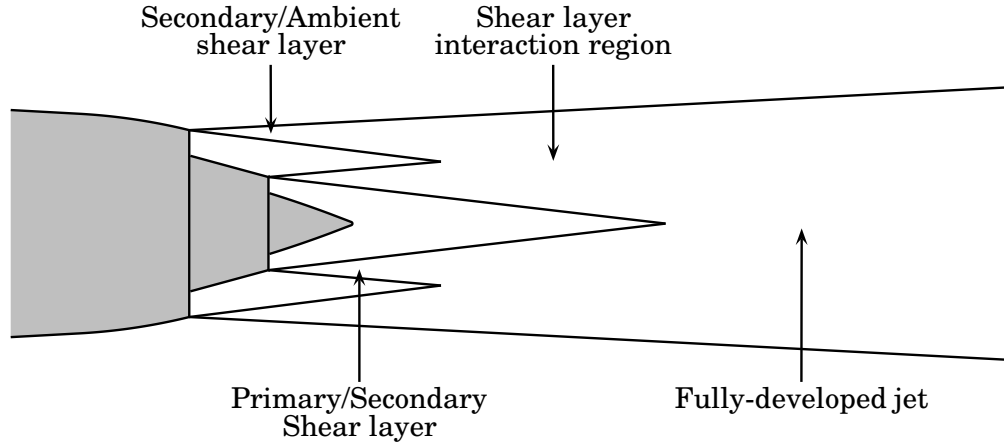


Figure 5.2: Flow regions in a developing dual-stream jet

5.4.2 Work and Results

Two simulations are reported in the paper. In the first simulation performed (*case 1*), the turbulence kinetic energy in the shear layer between the bypass stream and the stagnant surroundings was found to be significantly higher than in a RANS simulation for the same nozzle geometry and flow conditions made prior to the LES study described in the paper. These high energy levels were believed to be caused by the fact that the turbulent mixing was very low close to the nozzle exit, which led to the generation of large structures; hence a more violent mixing process occurred once initiated. After evaluation of the first simulation, three modifications were made to the simulation set-up for reasons discussed in more detail in Paper IV. A second simulation (*case 2*) was made with this modified set-up. The effects on the predicted flowfield and the acoustic signature are described in the paper. The modifications made in the computational set-up were the following:

1. The SGS filter width was decreased from $\Delta = (\Delta_1 \Delta_2 \Delta_3)^{(1/3)}$ in the first simulation to $\Delta = \min(\Delta_1, \Delta_2, \Delta_3)$ in the latter simulation with the objective of decreasing the SGS dissipation in the regions

close to the nozzle and hence improving the resolved mixing in the outer shear layer.

2. When the first case was run, it was found that selective artificial damping for density was needed to ensure stability in the simulation. In the modified computational set-up, this artificial damping was replaced with a TVD limiter for density. This results in a less dissipative scheme and thus improves the numerical accuracy. This change had some positive effects on the shear layer flow.
3. In the first case, constant values of P_0 were specified at the nozzle inlet boundaries for both the core flow and the bypass flow. In the latter case, the inlet total pressures were given as profiles, $P_0 = P_0(r)$, which increased the boundary layer thickness in this simulation.

The modifications in the computational set-up resulted in a shear layer development that seemed to be more physically correct.

The computational domain is divided into three parts: a high-resolution LES region near the nozzle walls, a medium dense LES region and a coarse LES region. In the medium-resolution region the computational cells are approximately equidistant and the maximum size of the cells is chosen such that the mesh will support acoustic waves of frequencies up to 15 kHz, using 8 cells per wavelength. In the high-resolution LES region, the cells are twice as fine in each direction as in the medium dense region and the cells are stretched somewhat towards the nozzle walls. The cells in the coarse LES region are twice as wide in each direction as those in the medium-resolution LES region. The computational domain was discretized using a block-structured boundary-fitted mesh with 159 mesh blocks and approximately 2.0×10^7 nodes. It should be noted that both simulations were made without increasing the viscosity or scaling down the geometry, which means that, although 2.0×10^7 nodes were used, it is a rather coarse LES. Wall functions were thus used to represent the near nozzle flow where needed.

Entrainment velocities at the coarse LES domain boundary are obtained by using a 2D extension of the domain representing the flow outside the LES domain.

The shear layer between the two streams was found to be very thin in both simulations. The predicted low degree of mixing for this shear layer is believed to be physically correct and caused by a combination of stabilizing factors. First of all, the velocity and momentum ratios are reversed, i.e. the velocity in the primary stream is higher than the velocity in the secondary stream, but, owing to temperature differences,

the momentum flux is lower in the primary stream than in the secondary stream. This might have a stabilizing effect on the shear layer between the two streams. Moreover, the flow in the initial part of the inner shear layer is not fully axial but has a negative velocity component in the radial direction, which makes the shear layer concave. This curvature of the shear layer might also have a stabilizing effect on the flow.

High levels of turbulence kinetic energy found in the outer shear layer in the first simulation resulted in significantly higher predicted sound pressure levels in that simulation than in the subsequent simulation.

5.4.3 Comments

It should be noted that none of the simulations presented in this paper have been validated against experimental data. Nevertheless, it is believed that the latter of the two simulations is more physically correct than the first one. In either case, the results show that changes in the initial mixing process have great effects on the radiated sound and that it therefore is important to capture the physics in the initial jet region to be able to predict the sound directivity. The peak sound pressure level was the same in both cases, which indicates that the initial mixing process is not as important for the rear arc observers as for the forward arc observers. This is, however, probably an effect of the fact that the changes observed in the shear-layer region did not affect the flow in the potential core region a great deal.

The transition process occurring in the initial shear layers in the simulations reported in Paper IV should probably not be as pronounced at the simulated Reynolds number as seen in the results. The shear-layer flow could probably be correctly represented by resolving the turbulent boundary layers on the nozzle walls and thus accomplish a direct transition from a turbulent boundary layer to a turbulent shear layer. Resolving the boundary layers place extreme demands for mesh resolution in the near-nozzle region and hence the presence of a transition process close to the nozzle is more or less inevitable. However, since the study reported in this paper shows that the axial extent of the transition region has significant effects on the predicted sound pressure levels, it is desirable to keep transition region as short as possible.

Due to the computational costs of performing a large-eddy simulation, the individual effects of the changes made to the computational set-up have not been investigated in detail. It is, however, believed that the change of filter width representation and the introduction of

the TVD limiter are the changes that made the most significant contribution to the final results.

The stepwise increased mesh resolution and entrainment boundary condition introduced in this study should be added to the list of uncertainties mentioned in the comments for Paper I, Section 5.1.3.

Chapter 6

Further Investigation of the CoJeN Coaxial Nozzle/Jet Configuration

THIS chapter gives preliminary results of an investigation of the effects of mixing enhancement devices on the development of the primary shear layer. Figure (6.1) shows how vortex generators have been placed on the outer wall in the near-lip region of the inner nozzle in order to break up structures in the inner shear layer close to the nozzle lip. Eight pairs of vortex generator blades are equally distributed around the nozzle wall. The vortex generators protrude into the boundary layer on the outside of the primary nozzle and thus break up near-wall ring-shaped vortex structures. Figures (6.2) and (6.3) depict density iso-surfaces that show shear layer structures in the inner shear layer and density contours, respectively. The base-nozzle flowfield corresponds to the second simulation reported in Paper IV (*case 2*), and the vortex-generator-nozzle results were obtained using the same computational set-up as for the base-nozzle case except that the mixing enhancement devices were added. As can be seen in the figures the vortex generators cause significant changes in the predicted flowfield. The result is a more violent mixing process close to the nozzle and thus a more rapidly growing shear layer and a somewhat shorter potential core region. It should be noted that these results are preliminary and that the observed changes in the shear flow behavior are changes from a simulation that has not been validated against measurements. Furthermore, the effects would probably not be as pronounced for a real jet engine since the simulations are made for ideal isolated nozzle geometries. At this stage no investigation has been done of the actual changes to the flow statistics by the mixing enhancement devices. This is obviously of great interest and is recommended as a continuation of

these preliminary studies.

The reason for placing the vortex generators on the inner nozzle was that studies of mixing enhancement devices such as chevrons and tabs have shown that affecting the inner shear layer of a coaxial jet configuration gives the best results in terms of reduced radiated sound (Saiyed *et al.*, 2000; Page *et al.*, 2002). One possible reason for this effect is that any increase in sound radiation that is caused by more violent mixing close to the nozzle might be shielded by the secondary shear layer and thus be less prominent than if the mixing devices were placed in the secondary shear layer. However, the changes in the near-nozzle shear layer might influence the downstream flow and cause significant changes in the flow in the regions where the potential core ends. This in turn might have effects on the radiated sound. Preliminary results indicate that the vortex generators can give a noise reduction for the rear-arc observers. However, currently, no fully trusted qualitative measures of achieved noise reduction have been obtained. Further investigations are needed to establish the actual effects of the mixing enhancement on the radiated sound. Note that, in order to be correctly assessed, achieved noise abatement must be related to the corresponding thrust penalty owing to momentum loss.

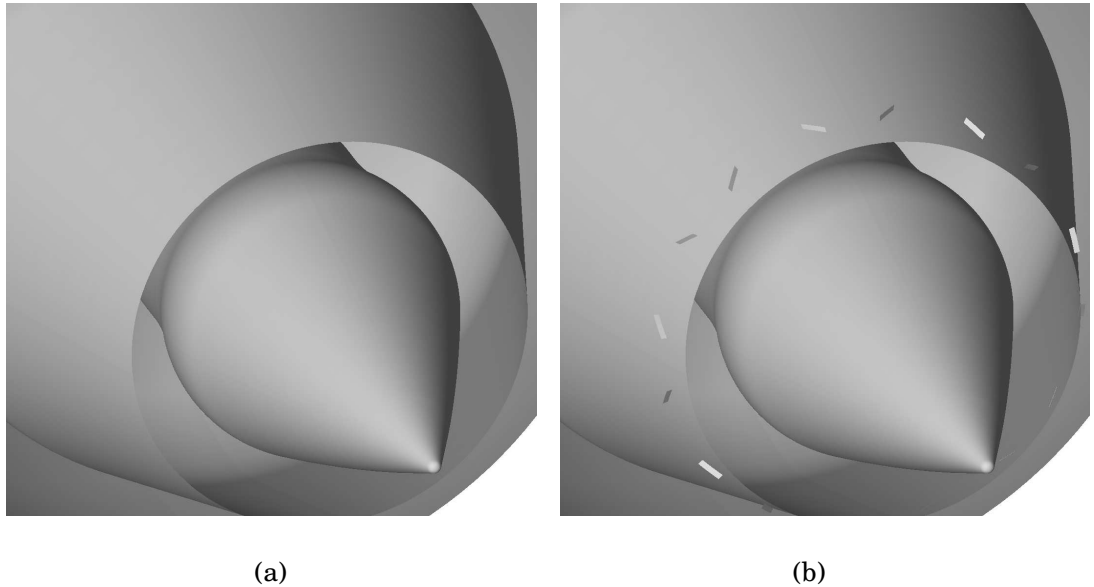


Figure 6.1: The CoJeN coaxial nozzle base configuration (a) and with the application of vortex generators (b)

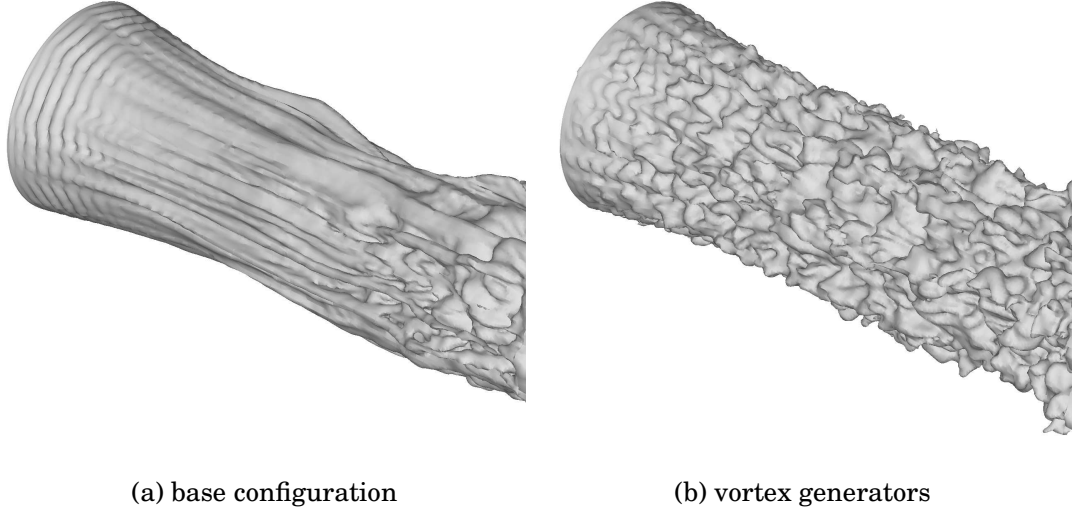


Figure 6.2: The primary shear layer in the near-nozzle region visualized by density iso-surfaces

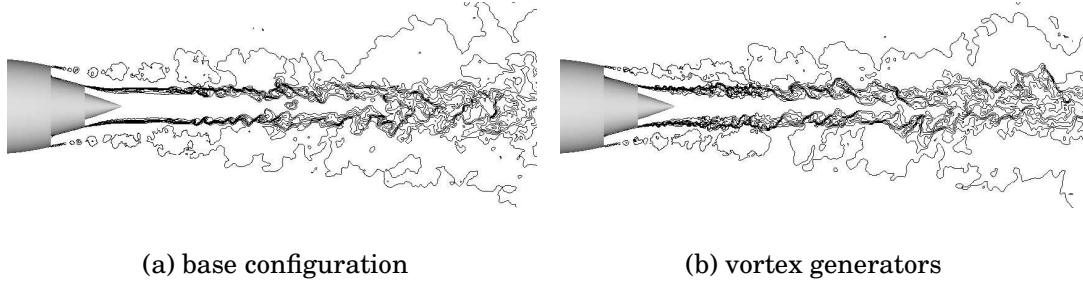


Figure 6.3: Instantaneous density contours

6.1 Vortex Generator Implementation

In this preliminary study a methodology for simulation of vortex generators reported by Wallin & Eriksson (2006) was implemented in the LES solver. Using the model proposed by Wallin & Eriksson (2006), which is a continuation of the work by Eriksson (1992) and Baralon *et al.* (1997), volume sources are added to the Navier-Stokes equations so as to represent the effect of the vortex generators on the flowfield, i.e. the vortex generator surfaces are fictitious and no changes are made in the mesh. The vortex generators does not have to be aligned with the mesh, see Figure (6.4), and the blade thickness is neglected.

In the present work the corner nodes of each vortex generator surfaces have been defined by a set of grid nodes in the mesh used for

the simulations reported in Paper IV and each surface is divided into a number of surface segments defined by grid lines axially along the nozzle wall, local I -direction in Figure (6.4), and radial grid lines normal to the nozzle wall, local J -direction. For each surface segment a local flow state at the vortex generator mean surface is estimated by linear interpolation involving two adjacent cells in the local K -direction, see Figure (6.4). Using these interpolated values surface forces are calculated using a flow tangency condition related to the rate of change of the local normal force F as

$$\frac{\partial F}{\partial t} = -\lambda (\rho u_i n_i)^{VGS} \quad (6.1)$$

where superscript VGS denotes vortex generator surface, n_i is the surface normal and λ is a constant proportional to the inverse of a local time scale squared. In the work reported in this chapter the solver time step has been chosen as a representation of the local time scale. The calculated surface forces are re-distributed, in the form of volume forces, to the neighboring cells from which the flow properties were interpolated and thus the net effect of the vortex generators on the flow is achieved without making changes to the nozzle geometry. Although the vortex generator blades themselves are very simple it would take a lot of effort to generate a high-quality mesh including the blades. Therefore, this methodology would probably be suitable for conceptual studies of mixing enhancement devices using LES. It could even, if implemented in a less computational demanding RANS based solver, as done by Wallin & Eriksson (2006), be used for parameter studies and optimization of mixing enhancement devices.

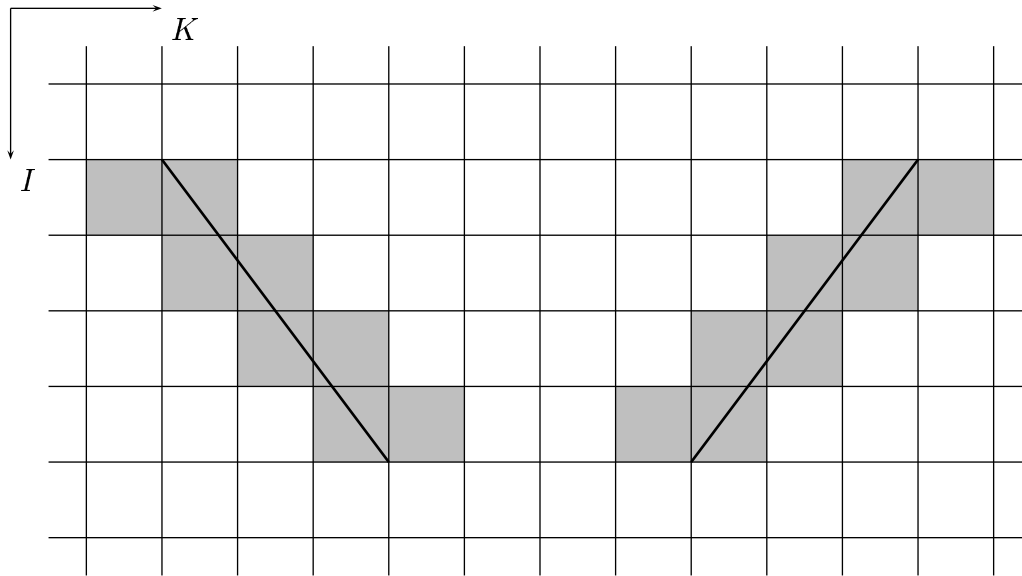


Figure 6.4: Schematic illustration of a vortex generator pair (cf. Figure (6.1(b))). Volume forces are distributed to the two cells closest to the vortex generators in each row in the local K -direction (highlighted cells).

Chapter 7

Concluding Remarks

SUBSONIC turbulent jets were simulated using compressible large-eddy simulation. A single-stream jet at Mach 0.75 and Reynolds number 5.0×10^4 was simulated at two jet-to-ambient temperature ratios. One simulation was made for isothermal jet conditions and one for a jet temperature twice that of the surrounding fluid. In the continued work, a high-subsonic dual-stream jet with a core flow temperature approximately three times the bypass flow temperature was simulated. The temperature of the bypass stream was equal to the ambient temperature and the jet Mach number was approximately 0.9 for both streams. All simulations were made with the nozzle geometry included in the computational domain. The acoustic field was extended to the far field using Kirchhoff surface integration. For the single-stream jet, predicted near-field and far-field data were found to be in good agreement with available measurements. No experimental data for validation of the LES predictions for the coaxial jet configuration are available at this stage of the CoJeN project, which means that the results obtained for the dual-stream jet have not been validated against measurements.

7.1 Single-Stream Jet

The maximum levels of turbulence intensities are well captured for both the isothermal and the heated jet. Furthermore, the turbulence anisotropy is correctly predicted in both jets. The initial jet spreading and the potential core lengths are not predicted correctly, however. Overpredicted $u'v'$ correlations close to the nozzle exit indicate that the degree of mixing is too high, which makes the transition process too efficient, hence shortening the length of the potential region.

Two-point space-time correlations obtained in the shear layer at an axial location corresponding to the end of the potential core were in good agreement with experiments by Jordan & Gervais (2003). Estimates of integral length scales, integral time scales and eddy convection velocities were obtained from the predicted correlation data. The agreement with corresponding quantities obtained from the measured data was satisfying.

Only small differences could be identified in correlations and integral scales obtained for the isothermal and the heated jets.

The sensitivity of flowfield and radiated sound to changes in inflow conditions, subgrid-scale quantities and Reynolds number was investigated, and it seems that the flow statistics in the near-nozzle region is rather insensitive to these modifications. In particular, none of the changes from the computational set-up used in the reference simulation gave a potential core length that was in better agreement with the measured data.

The mesh used for the single-stream jet simulations fails to capture acoustic waves of Strouhal numbers higher than $St \simeq 1.2$. This results in a rapid fall-off of the spectrum amplitudes for higher Strouhal numbers. The lower part of the spectra is accurately captured, however.

The predicted sound pressure levels for all observers are within a 3.0 dB deviation from the measured levels and, for most observers, within a deviation of 1.0 dB.

Capturing the lower part of the spectra seems to be sufficient to represent the major portion of the radiated sound. However, if the aim were to capture the high frequencies of the sound spectra, a higher mesh resolution and probably higher-order schemes, as e.g. the DRP scheme proposed by Tam & Webb (1993), would be desirable.

Both measurements and predictions show slightly higher SPL for the heated jet. The increase in SPL is however too small to be able to draw conclusions on the effects of the increased temperature ratio on the radiated sound.

Introducing flux correction terms that approximate the effects of the Leonard stresses appearing in the spatially filtered governing equations gave better representation of the high-frequency content of the radiated sound. This improves the sound predictions

for the forward arc observers. It should be mentioned, however, that this improvement comes with a penalty in calculation cost.

7.2 Coaxial Jet

The subgrid-scale filter width was decreased from $\Delta = (\Delta_1 \Delta_2 \Delta_3)^{1/3}$ to $\Delta = \min(\Delta_1, \Delta_2, \Delta_3)$ with the objective of decreasing the SGS dissipation in the regions close to the nozzle and hence improving the resolved mixing in the outer shear layer. Furthermore, the accuracy of the numerical scheme was improved by replacing the artificial selective damping needed to obtain stability in the first simulation (*case 1*) with a TVD limiter for density in the second simulation (*case 2*). Owing to a decrease in numerical dissipation this further improved the mixing in the outer shear layer.

High levels of turbulence kinetic energy found in the outer shear layer in *case 1* resulted in significantly higher predicted sound pressure levels in *case 1* than in *case 2*. This shows that changes in the initial mixing process have pronounced effects on the radiated sound and that it is thus important to capture the physics in the initial jet region to be able to predict the sound directivity.

Placing vortex generators on the outside wall of the primary nozzle led to the break up of ring-shaped vortex structures in the primary shear layer and thus significantly changed the flow in this region.

7.3 Recommendations for Future Work

The majority of the jet noise studies that have used LES deal with moderate Reynolds number isothermal single-stream jets with a stagnant surrounding fluid. Some studies have recently been published in which more complex nozzle geometries and heated jets at high Reynolds number have been considered. Although these studies are steps forward towards a prediction of a realistic jet engine exhaust noise, there is still quite a lot of work to be done to reach conditions representative for a jet engine at take-off. For instance, if, as in the present study, a surface integration approach is utilized to obtain the acoustic signature, the integral method must cope with high mean flow velocities in the fluid surrounding the jet exhaust flow. For example, the freestream Mach number in an operating point representative for aircraft noise certification conditions is in the range of 0.25-0.30. Most studies reported fo-

cuses primarily on noise generated in isolated jets. Therefore, it would be interesting to study installation effects such as jet pylon interaction in terms of changes to shear layer structures and effects on sound radiation, as done in e.g. Hunter & Thomas (2003). Another issue that must be studied in more detail in the progress towards more realistic flow conditions is the effects on the development of the shear layer and on the sound generation of upstream disturbances in the form of unsteady wakes generated by a succession of wake-blade interactions in the core and bypass ducts.

Noise suppression techniques have to be studied in more detail. As mentioned in Chapter 6, it would be interesting to perform a parametric study of the shape, size and location of mixing enhancement devices such as for example chevrons, tabs and vortex generators in order to optimize the nozzle design for achievement of as high noise reduction as possible with low thrust penalty.

For the development of design tools for industrial use, research of a more fundamental nature needs must be done, such as for example the work reported by Bogey *et al.* (2003) where correlations of peaks in the far-field pressure signals and events in the jet flow were investigated.

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Appendix A

Characteristic Variables

A purely inviscid flow is governed by the Euler equations, i.e. the inviscid part of the Navier-Stokes equations, which, if the flow is smooth, can be rewritten in a quasi-linear form as

$$\frac{\partial Q}{\partial t} + \frac{\partial \mathcal{F}_i}{\partial Q} \frac{\partial Q}{\partial x_i} = 0 \quad (\text{A.1})$$

Matrices $\frac{\partial \mathcal{F}_i}{\partial Q}$ are often referred to as flux Jacobians. For local analysis, the flux Jacobians may be regarded as constants, and thus a linear relation can be obtained according to

$$\frac{\partial Q}{\partial t} + \left(\frac{\partial \mathcal{F}_i}{\partial Q} \right)_0 \frac{\partial Q}{\partial x_i} = 0 \quad (\text{A.2})$$

In Eqn. (A.2), subscript 0 denotes the reference state for which the flux Jacobians have been evaluated. Assuming that the perturbations correspond to planar waves in space with the plane normally aligned with the vector $(\alpha_1, \alpha_2, \alpha_3)$ for which $\alpha_i \alpha_i = 1$, Eqn. (A.2) can be rewritten as

$$\frac{\partial Q}{\partial t} + \tilde{A}_0 \frac{\partial Q}{\partial \xi} = 0 \quad (\text{A.3})$$

where

$$\tilde{A}_0 = \alpha_i \left(\frac{\partial \mathcal{F}_i}{\partial Q} \right)_0 \quad (\text{A.4})$$

is a linear combination of the flux Jacobians and

$$\xi = \alpha_i x_i \quad (\text{A.5})$$

is a coordinate aligned with the wave plane normal vector. Eqn. (A.3) is a coupled system of one-dimensional linear equations describing the propagation of waves along the ξ -axis. Matrices T and T^{-1} are found

such that matrix $\tilde{A}_0$ is diagonalized using a standard eigen-decomposition procedure

$$T\tilde{A}_0T^{-1} = \Lambda = \begin{bmatrix} \lambda_1 & 0 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 & 0 \\ 0 & 0 & \lambda_3 & 0 & 0 \\ 0 & 0 & 0 & \lambda_4 & 0 \\ 0 & 0 & 0 & 0 & \lambda_5 \end{bmatrix} \quad (\text{A.6})$$

The eigenvalues $(\lambda_1, \dots, \lambda_5)$ can be calculated analytically as

$$\begin{aligned} \lambda_1 &= \lambda_2 = \lambda_3 = \alpha_i u_i \\ \lambda_4 &= \lambda_1 + c\sqrt{\alpha_i \alpha_i} \\ \lambda_5 &= \lambda_1 - c\sqrt{\alpha_i \alpha_i} \end{aligned} \quad (\text{A.7})$$

where c is the speed of sound. Introducing the characteristic variables defined as

$$W = T^{-1}Q \quad (\text{A.8})$$

the system of linear equations given by Eqns. (A.3) can be written as a non-coupled set of linear transport equations

$$\frac{\partial W}{\partial t} + \Lambda \frac{\partial W}{\partial \xi} = 0 \quad (\text{A.9})$$

Each of the five components, w_i , of the W vector is a characteristic variable transported in a direction aligned with the ξ -axis with the characteristic speed described by the corresponding eigenvalue, λ_i . The first of the five characteristic variables may be interpreted physically as an entropy wave, the following two as vorticity waves and the last two as acoustic waves.

Appendix B

Physical and Computational Space

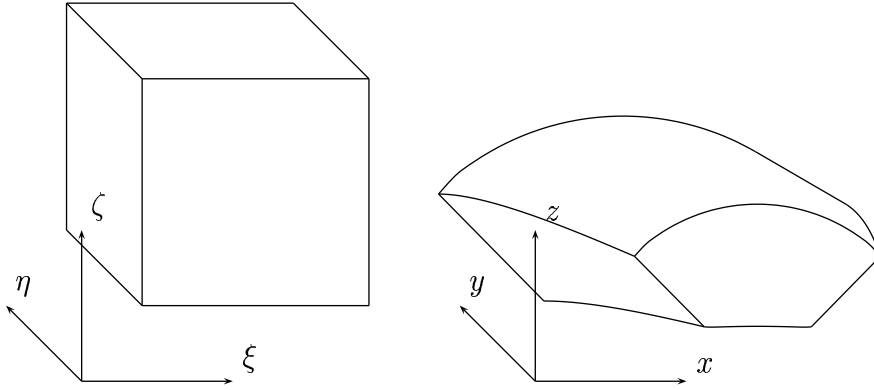


Figure B.1: A domain depicted in computational space and physical space

AN arbitrary mapping between the computational domain coordinates, i.e. (ξ, η, ζ) , and the coordinates in the physical domain, i.e. (x, y, z) , or vice versa, in three dimensions can be expressed as follows

$$\begin{cases} x = x(\xi, \eta, \zeta) \\ y = y(\xi, \eta, \zeta) \\ z = z(\xi, \eta, \zeta) \end{cases}, \begin{cases} \xi = \xi(x, y, z) \\ \eta = \eta(x, y, z) \\ \zeta = \zeta(x, y, z) \end{cases} \quad (\text{B.1})$$

Differentiating the mapping from the physical to the computational domain gives

$$\begin{aligned} dx &= x_\xi d\xi + x_\eta d\eta + x_\zeta d\zeta \\ dy &= y_\xi d\xi + y_\eta d\eta + y_\zeta d\zeta \\ dz &= z_\xi d\xi + z_\eta d\eta + z_\zeta d\zeta \end{aligned} \tag{B.2}$$

$$\begin{bmatrix} dx \\ dy \\ dz \end{bmatrix} = \begin{bmatrix} x_\xi & x_\eta & x_\zeta \\ y_\xi & y_\eta & y_\zeta \\ z_\xi & z_\eta & z_\zeta \end{bmatrix} \begin{bmatrix} d\xi \\ d\eta \\ d\zeta \end{bmatrix} = A \begin{bmatrix} d\xi \\ d\eta \\ d\zeta \end{bmatrix}$$

where subscripts ξ , η and ζ denote differentiation with respect to the coordinates in computational space, e.g. $x_\xi = \frac{\partial x}{\partial \xi}$. Differentiating the mapping from the computational to the physical domain gives

$$\begin{aligned} d\xi &= \xi_x dx + \xi_y dy + \xi_z dz \\ d\eta &= \eta_x dx + \eta_y dy + \eta_z dz \\ d\zeta &= \zeta_x dx + \zeta_y dy + \zeta_z dz \end{aligned} \tag{B.3}$$

$$\begin{bmatrix} d\xi \\ d\eta \\ d\zeta \end{bmatrix} = \begin{bmatrix} \xi_x & \xi_y & \xi_z \\ \eta_x & \eta_y & \eta_z \\ \zeta_x & \zeta_y & \zeta_z \end{bmatrix} \begin{bmatrix} dx \\ dy \\ dz \end{bmatrix} = A^{-1} \begin{bmatrix} dx \\ dy \\ dz \end{bmatrix}$$

where the inverse of A can be calculated from A as follows using the cofactor method

$$[A^{-1}]_{ij} = \frac{(-1)^{i+j}}{\det(A)} D_{ji} \tag{B.4}$$

where D_{ij} is a sub-determinant of A obtained by deleting the i^{th} row and the j^{th} column. The inverse of A can now be expressed in terms of coordinates in the physical domain differentiated with respect to the coordinates in the computational domain as follows

$$A^{-1} = \frac{1}{\det(A)} \begin{bmatrix} (y_\eta z_\zeta - y_\zeta z_\eta) & (x_\zeta z_\eta - x_\eta z_\zeta) & (x_\eta y_\zeta - x_\zeta y_\eta) \\ (y_\zeta z_\xi - y_\xi z_\zeta) & (x_\xi z_\zeta - x_\zeta z_\xi) & (x_\zeta y_\xi - x_\xi y_\zeta) \\ (y_\xi z_\eta - y_\eta z_\xi) & (x_\eta z_\xi - x_\xi z_\eta) & (x_\xi y_\eta - x_\eta y_\xi) \end{bmatrix} \tag{B.5}$$

where the determinant of A is calculated as

$$\det(A) = x_\xi (y_\eta z_\zeta - y_\zeta z_\eta) + y_\xi (x_\zeta z_\eta - x_\eta z_\zeta) + z_\xi (x_\zeta y_\eta - x_\eta y_\zeta) \tag{B.6}$$

Differentiating a flow variable, Φ , with respect to ξ , η and ζ implies

$$\begin{aligned} d\Phi &= \frac{\partial \Phi}{\partial \xi} d\xi + \frac{\partial \Phi}{\partial \eta} d\eta + \frac{\partial \Phi}{\partial \zeta} d\zeta = \begin{bmatrix} \frac{\partial \Phi}{\partial \xi} & \frac{\partial \Phi}{\partial \eta} & \frac{\partial \Phi}{\partial \zeta} \end{bmatrix} \begin{bmatrix} d\xi \\ d\eta \\ d\zeta \end{bmatrix} \\ &= \begin{bmatrix} \frac{\partial \Phi}{\partial \xi} & \frac{\partial \Phi}{\partial \eta} & \frac{\partial \Phi}{\partial \zeta} \end{bmatrix} A^{-1} \begin{bmatrix} dx \\ dy \\ dz \end{bmatrix} \end{aligned} \tag{B.7}$$

APPENDIX B. PHYSICAL AND COMPUTATIONAL SPACE

Differentiating the same flow variable with respect to x , y and z gives

$$d\Phi = \frac{\partial\Phi}{\partial x}dx + \frac{\partial\Phi}{\partial y}dy + \frac{\partial\Phi}{\partial z}dz = \begin{bmatrix} \frac{\partial\Phi}{\partial x} & \frac{\partial\Phi}{\partial y} & \frac{\partial\Phi}{\partial z} \end{bmatrix} \begin{bmatrix} dx \\ dy \\ dz \end{bmatrix} \quad (\text{B.8})$$

By identification of terms on the right-hand side of Eqns. (B.7) and (B.8), respectively, a relation between spatial derivatives in computational space and physical space can be obtained as

$$\begin{bmatrix} \frac{\partial\Phi}{\partial x} & \frac{\partial\Phi}{\partial y} & \frac{\partial\Phi}{\partial z} \end{bmatrix} = \begin{bmatrix} \frac{\partial\Phi}{\partial \xi} & \frac{\partial\Phi}{\partial \eta} & \frac{\partial\Phi}{\partial \zeta} \end{bmatrix} A^{-1} \quad (\text{B.9})$$

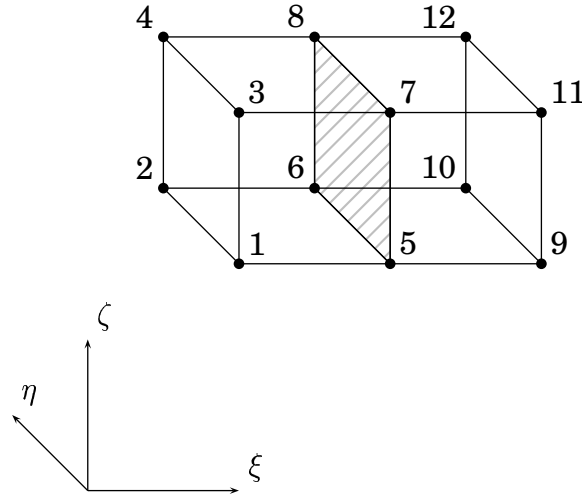


Figure B.2: Two adjacent cells in computational space, i.e. (ξ, η, ζ) -space, defined by corner points 1–12

Using the twelve corner points defining two adjacent cells, see Figure (B.2), derivatives of the physical space coordinates with respect to the coordinates in computational space at the cell face can be obtained as follows.

$$\begin{aligned}\frac{\partial x}{\partial \xi} &= \frac{1}{8} \left[(x_9 + x_{10} + x_{11} + x_{12}) - (x_1 + x_2 + x_3 + x_4) \right] \\ \frac{\partial y}{\partial \xi} &= \frac{1}{8} \left[(y_9 + y_{10} + y_{11} + y_{12}) - (y_1 + y_2 + y_3 + y_4) \right] \\ \frac{\partial z}{\partial \xi} &= \frac{1}{8} \left[(z_9 + z_{10} + z_{11} + z_{12}) - (z_1 + z_2 + z_3 + z_4) \right]\end{aligned}$$

$$\begin{aligned}\frac{\partial x}{\partial \eta} &= \frac{1}{2} \left[(x_6 + x_8) - (x_5 + x_7) \right] \\ \frac{\partial y}{\partial \eta} &= \frac{1}{2} \left[(y_6 + y_8) - (y_5 + y_7) \right] \\ \frac{\partial z}{\partial \eta} &= \frac{1}{2} \left[(z_6 + z_8) - (z_5 + z_7) \right]\end{aligned} \tag{B.10}$$

$$\begin{aligned}\frac{\partial x}{\partial \zeta} &= \frac{1}{2} \left[(x_7 + x_8) - (x_5 + x_6) \right] \\ \frac{\partial y}{\partial \zeta} &= \frac{1}{2} \left[(y_7 + y_8) - (y_5 + y_6) \right] \\ \frac{\partial z}{\partial \zeta} &= \frac{1}{2} \left[(z_7 + z_8) - (z_5 + z_6) \right]\end{aligned}$$

Appendix C

MPI Implementation

THE MPI implementation has been made such that none of the processes need to have access to the entire calculation problem so as to minimize the memory use. A user-defined problem-split definition read by all processes when the code is executed gives instructions as to which mesh blocks belongs to which process. Each process generates a local mesh and a local set of variables based on the split instructions. At the same time as the local problem is formed, a pointer structure between the local and global calculation mesh blocks is generated. This makes it possible for each process to distinguish between local mesh block interfaces and interfaces between local mesh blocks and mesh blocks in other processes. Furthermore, the pointer information is used to decide whether or not boundary condition information and instructions for local evaluation such as Kirchhoff surface integration and sample routines given in the problem set-up are relevant for a given process. An additional pointer structure is generated before the calculation process is started. This pointer structure is used to collect interface information such that it is possible to perform all send and receive operations between two processes in one MPI communication, see Figure (C.1). Each process stores data that are to be sent to other processes in dedicated arrays. The pointer information gives certain process instructions about where which data shall be stored. If a process has more than one interface common with another process, the data for these interfaces are stored as a sequence in the array, which makes it possible to send all the interface data as one record. The received data are stored in a corresponding receive array.

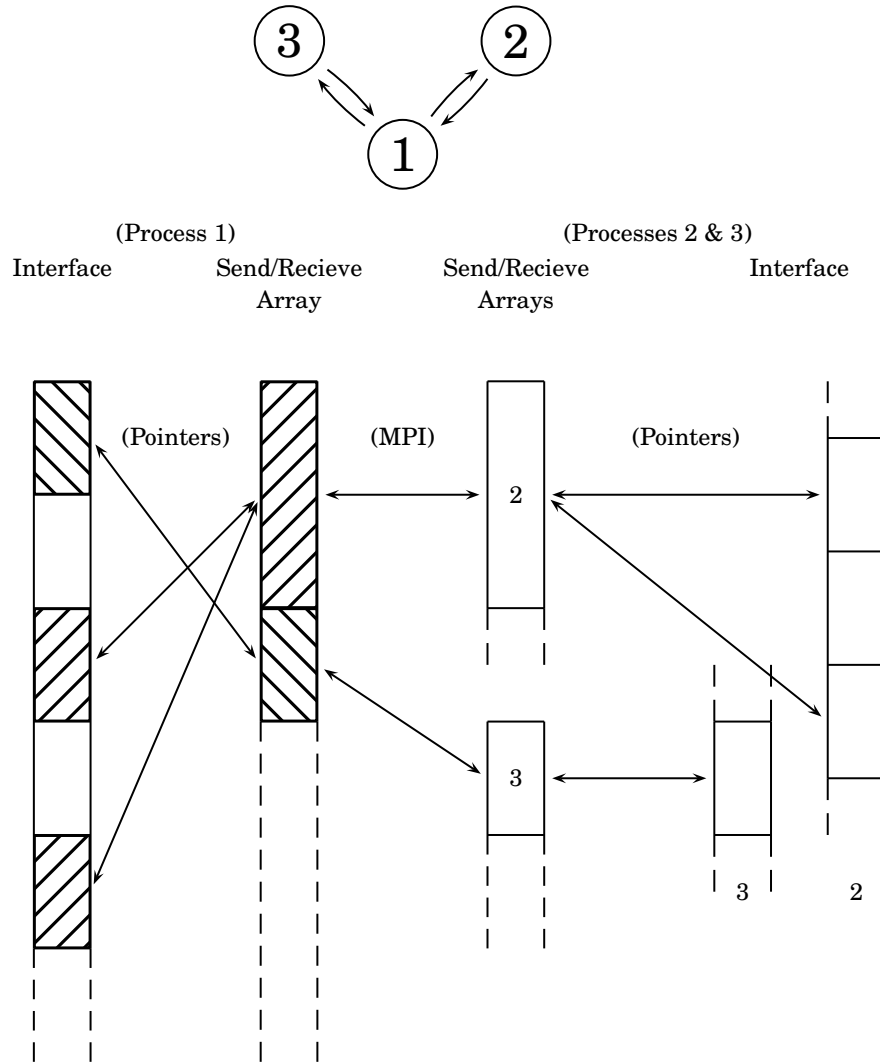


Figure C.1: Send and receive structures. A schematic illustration of the MPI communication between three processes with common mesh block interface areas.

Paper I

Investigation of an Isothermal Mach 0.75 Jet and its Radiated Sound Using Large-Eddy Simulation and Kirchhoff Surface Integration

By

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Investigation of an isothermal Mach 0.75 jet and its radiated sound using large-eddy simulation and Kirchhoff surface integration

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Abstract

A large-eddy simulation (LES) of a compressible nozzle/jet configuration has been carried out. An isothermal Mach 0.75 jet was simulated. The Reynolds number based on the jet velocity at the nozzle exit plane and the nozzle diameter was 5.0×10^4 . The Favre filtered Navier–Stokes equations were solved using a finite volume method solver with a low-dissipation third-order upwind scheme for the convective fluxes, a second-order centered difference approach for the viscous fluxes and a three-stage second-order Runge–Kutta time marching technique. A compressible form of Smagorinsky's subgrid scale model was used for computation of the subgrid scale stresses. The computational domain was discretized using a block structured boundary fitted mesh with approximately 3.0×10^6 cells. The calculations were performed on a parallel computer, using message-passing interface (MPI). Absorbing boundary conditions based on characteristic variables were adopted for all free boundaries. Velocity components specified at the entrainment boundaries were estimated from a corresponding Reynolds averaged Navier–Stokes (RANS) calculation, which enabled the use of a rather narrow domain. In order to diminish disturbances caused by the outlet boundary, a buffer layer was added at the domain outlet. Kirchhoff surface integration using instantaneous pressure data from the LES was utilized to obtain far-field sound pressure levels in a number of observer locations. The predicted sound pressure levels were for all observer locations within a 3 dB deviation from the measured levels and for most observer locations within a 1 dB deviation. Aerodynamic results and predicted sound pressure levels are both in good agreement with experiments. Experimental data were provided by Laboratoire d'Etude Aérodynamiques, Poitiers, France [Jordan, P., Gervais, Y., Valière, J.-C., Foulon, H., 2002. Final results from single point measurements. Project deliverable D3.4, JEAN—EU 5th Framework Programme, G4RD-CT2000-00313, Laboratoire d'Etude Aérodynamiques, Poitiers; Jordan, P., Gervais, Y., Valière, J.-C., Foulon, H., 2002. Results from acoustic field measurements. Project deliverable D3.6, JEAN—EU 5th Framework Programme, G4RD-CT2000-00313, Laboratoire d'Etude Aérodynamiques, Poitiers; Jordan, P., Gervais, Y., 2003. Modeling self and shear noise mechanisms in anisotropic turbulence. In: The 9th AIAA/CEAS Aeroacoustics Conference. No. 8743 in AIAA 2003. Hilton Head, SC].

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1. Introduction

Restrictions of noise levels in the surroundings of airports have made the reduction of near ground operation noise an important issue for aircraft and engine manufacturers, and noise generation has now become an important design factor taken into consid-

eration early in the construction process. At take-off the main sources of noise are the propelling jet and the engine fan, of which the jet is usually the strongest noise source at full power. An EU project, JEAN (Jet Exhaust Aerodynamics and Noise), that focuses on investigating jet noise both numerically and experimentally and has a long term aim to improve existing noise predicting tools, was started in February 2001. The work to be presented has been done within this project.

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size turbulence in the noise source regions, see [Billson \(2004\)](#). This method is promising since simulations of high Reynolds number flows are possible with reasonable computational efforts. In contrast to a RANS calculation, where all turbulent scales in the flow are modeled and only a time-averaged flow field is obtained, DNS and LES directly provide information about turbulent quantities and sources of noise. To be able to improve existing CFD routines for more reliable results, we need an understanding of noise source mechanisms. Therefore, it is of great importance to perform more detailed calculations, such as DNS or LES, to get a more realistic picture of the flow physics. This is the main objective of the present work.

LES and DNS have been used for jet flow applications in a number of publications. These mostly study jets at moderate Reynolds number due to the high computational costs of performing simulations of a high Reynolds number jet. Many of these studies have been carried out in order to predict jet noise. However, as it is a free shear flow frequently occurring in both nature and industrial applications, the jet is interesting to study in itself. Some studies are thus pure investigations of flow phenomena.

The feasibility of using LES for both the flow field and the radiated sound from a high subsonic 6.5×10^4 Reynolds number jet has been discussed by [Bogey et al. \(2000a, 2001, 2003\)](#). In [Bogey et al. \(2000a, 2003\)](#), the acoustic field was obtained directly from the flow simulation. Noise generation mechanisms were found to be relatively independent of Reynolds number. In [Bogey et al. \(2001\)](#) Lighthill's acoustic analogy was used in combination with compressible LES to obtain the acoustic field. [Bogey and Bailly \(2003\)](#) investigated the effects of inflow conditions on the flow field and the radiated sound of a high Reynolds number, $Re_D = 4.0 \times 10^5$, Mach 0.9 jet. Both the flow development and the emitted sound were shown to depend appreciably on initial parameters. The effects of inflow conditions on the self-similar region were studied by [Boersma et al. \(1998\)](#) using DNS. The Reynolds number in this study was 2.4×10^3 . [Freund \(2001\)](#) investigated sources of sound in a Mach 0.9 jet at a Reynolds number of $Re_D = 3.6 \times 10^3$ using DNS. In this work the part of the Lighthill source that may radiate to the far-field was isolated using Fourier methods. It was found that the peak of the radiating source coincides with neither the peak of the total source nor the peak of turbulence kinetic energy. The flow field and the radiated sound of a supersonic

low Reynolds number jet (Mach 1.92, $Re_D = 2.0 \times 10^3$) was predicted using DNS by [Freund et al. \(2000\)](#). [DeBonis and Scott \(2002\)](#) used LES to obtain the flow field of a supersonic high Reynolds number jet (Mach 1.4, $Re_D = 1.2 \times 10^6$) from which two-point space-time correlations in the jet shear-layer were obtained. Turbulence scales obtained from the correlations were found to be in good agreement with theory. In this study the nozzle geometry was included in the calculation domain. [Shur et al. \(2003\)](#) made simulations of a cold Mach 0.9 jet at a Reynolds number of 1.0×10^4 . Radiated sound was successfully predicted using [Ffowcs Williams and Hawkings \(1969\)](#) surface integral formulation, which since the surface is stationary becomes identical to the formulation by [Curle \(1955\)](#). The simulation was conducted using only 5.0×10^5 cells. This work was done using the monotone-integrated LES (MILES) approach where the subgrid scale model is replaced by the numerical dissipation of the numerical method used. [Tucker \(2004\)](#) investigated the effects of co-flow and swirl on the flow in the initial jet region and the radiated noise of a high subsonic jet (Mach 0.9, $Re_D = 1.0 \times 10^4$) using a hybrid RANS/MILES approach. [Le Ribault et al. \(1999\)](#) performed large-eddy simulations of a plane jet at two Reynolds numbers, $Re = 3.0 \times 10^3$ and $Re = 3.0 \times 10^4$. Simulations were performed for both Reynolds numbers using different subgrid scale models to investigate the ability of the models to capture the jet flow physics. [Zhao et al. \(2001\)](#) conducted an LES of a Mach 0.9 jet at Reynolds number 3.6×10^3 and a jet at Mach 0.4 and a Reynolds number of 5.0×10^3 . In this study, radiated sound was obtained both directly from the LES and by using Kirchhoff surface integration. The effect on the radiated sound of the subgrid scale model was investigated. It was found that using a mixed subgrid-scale model resulted in both higher turbulence levels and sound levels. [Rembold et al. \(2002\)](#) investigated the flow field of a rectangular jet at Mach 0.5 and a Reynolds number of 5.0×10^3 using DNS.

In the present study an LES of a Mach 0.75 nozzle/jet configuration has been performed (see [Table 1](#)). The Reynolds number based on the nozzle exit diameter and the jet velocity at the nozzle exit plane, Re_D , was 5.0×10^4 . The Reynolds number in the measurements ([Jordan et al., 2002a,b; Jordan and Gervais, 2003; Power et al., 2004](#)), used for comparison and validation of the simulations, was approximately one million. Such a high Reynolds number probably means that the scales that need to be resolved are too small. Thus the

Table 1
Flow properties

U/c_∞	T/T_∞	P_∞ (Pa)	ρ_∞ (kgm ⁻³)	c_∞ (ms ⁻¹)	U_∞ (ms ⁻¹)	T_∞ (K)	T_0 (K)	Re_D^a
0.75	1.0	101300	1.22556	340.174	0.0	288.0	320.4	5.0×10^4

^a The Reynolds number was decreased using modified viscosity.

Reynolds number in our LES was decreased with the assumption that the flow is only weakly Reynolds number dependent. A nozzle geometry is included in the calculation domain corresponding to the last contraction of the nozzle configuration in the experimental setup used for the measurements by Jordan et al. An isothermal jet is simulated, i.e. the static temperature in the nozzle exit plane, T_j , is equal to the static temperature of the ambient air, T_∞ . For validation of the LES results, the time-averaged flow is compared with experimental data provided by Laboratoire d'Etude Aérodynamiques, Poitiers, France (Jordan et al., 2002a).

Two-point space-time correlations are obtained for a few locations in the shear-layer, and integral length and time scales and eddy convection velocities are evaluated. Two-point measurements in the shear-layer at $x = L_c$ and $x = 0.75L_c$ done by Jordan and Gervais (2003) are used for comparison.

Evaluation is made of far-field sound pressure levels using Kirchhoff surface integration. Predicted sound pressure levels are compared with experimental results (Jordan et al., 2002b).

2. Governing equations

The equations solved are the spatially Favre filtered continuity, momentum and energy equations,

$$\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial (\bar{\rho} \tilde{u}_i)}{\partial x_i} = 0, \quad (1)$$

$$\frac{\partial (\bar{\rho} \tilde{u}_i)}{\partial t} + \frac{\partial (\bar{\rho} \tilde{u}_i \tilde{u}_j)}{\partial x_j} = -\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial \bar{\sigma}_{ij}}{\partial x_j} + \frac{\partial \tau_{ij}}{\partial x_j}, \quad (2)$$

$$\begin{aligned} \frac{\partial (\bar{\rho} \tilde{e}_0)}{\partial t} + \frac{\partial (\bar{\rho} \tilde{e}_0 \tilde{u}_j)}{\partial x_j} = & -\frac{\partial \bar{p} \tilde{u}_j}{\partial x_j} + \frac{\partial}{\partial x_j} \left(C_p \frac{\mu}{Pr} \frac{\partial \tilde{T}}{\partial x_j} + q_j \right) \\ & + \frac{\partial}{\partial x_j} (\tilde{u}_i (\bar{\sigma}_{ij} + \tau_{ij})), \end{aligned} \quad (3)$$

where $\bar{\sigma}_{ij}$ and τ_{ij} are the Favre filtered viscous stress tensor and subgrid scale viscous stress tensor, respectively. These are here defined as

$$\bar{\sigma}_{ij} = \mu \left(2\tilde{S}_{ij} - \frac{2}{3}\tilde{S}_{mm}\delta_{ij} \right), \quad (4)$$

$$\tau_{ij} = \mu t \left(2\tilde{S}_{ij} - \frac{2}{3}\tilde{S}_{mm}\delta_{ij} \right) - \frac{2}{3}\bar{\rho} k^{SGS} \delta_{ij}, \quad (5)$$

where k^{SGS} is the subgrid scale kinetic energy

$$k^{SGS} = C_I \Delta^2 \tilde{S}_{mn} \tilde{S}_{mn}, \quad (6)$$

μ_t the subgrid scale dynamic viscosity

$$\mu_t = C_R \bar{\rho} \Delta^2 \left(\tilde{S}_{mn} \tilde{S}_{mn} \right)^{0.5}, \quad (7)$$

and $\tilde{S}_{ij}$ is the Favre filtered strain rate tensor given by,

$$\tilde{S}_{ij} = \frac{1}{2} \left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i} \right) \quad (8)$$

The subgrid heat flux appearing in the Favre-filtered energy equation is modeled using a temperature gradient approach

$$q_j = C_p \frac{\mu_t}{Pr_t} \frac{\partial \tilde{T}}{\partial x_j}. \quad (9)$$

The filter-width used in Eqs. (6) and (7) is the local grid cell width, i.e. $\Delta = (\Delta_1 \Delta_2 \Delta_3)^{1/3}$. The subgrid scale model used in this work is the Smagorinsky part of the model proposed by Erlebacher et al. (1992) for compressible flows. The Smagorinsky model constants C_R and C_I were proposed by Erlebacher et al. (1992) to be

$$\begin{cases} C_R = 0.12, \\ C_I = 0.66. \end{cases} \quad (10)$$

The system of governing equations, Eqs. (1)–(3), is closed by making assumptions concerning the thermodynamics of the gas considered. It is assumed that the gas is thermally perfect, i.e. it obeys the gas law. Furthermore, the gas is assumed to be calorically perfect, which implies that internal energy and enthalpy are linear functions of temperature.

3. Kirchhoff surface integral formulation

Kirchhoff integration is a method for predicting the value of a property, Φ , governed by the wave equation, at a point outside a surface enclosing all generating structures (Lyrintzis, 1994).

$$\Phi(\mathbf{y}, t) = \frac{1}{4\pi} \int_S \left[\frac{\Phi}{r^2} \frac{\partial r}{\partial n} - \frac{1}{r} \frac{\partial \Phi}{\partial n} + \frac{1}{c_\infty r} \frac{\partial r}{\partial n} \frac{\partial \Phi}{\partial t} \right]_{\tau_r} dS(\mathbf{x}), \quad (11)$$

where τ_r denotes that the expression within brackets is to be evaluated at retarded time, i.e. emission time. τ_r is related to the observer evaluation time, t , as

$$\tau_r = t - \frac{r}{c_\infty}, \quad (12)$$

where r is the distance between a point on the surface, $\mathbf{x}$, and an observer location, $\mathbf{y}$. c_∞ is the speed of sound in the far-field region. The variable, Φ , to be evaluated, is in this case the surface pressure. S denotes the surface enclosing all sound generating structures and n denotes the direction normal to the surface. The surface, S , must be placed in a region where the flow is completely governed by a homogeneous linear wave equation with constant coefficients (Freund et al., 1996). More detail on the Kirchhoff surface integration method, can be found in e.g. Freund et al. (1996) and Lyrintzis (1994).

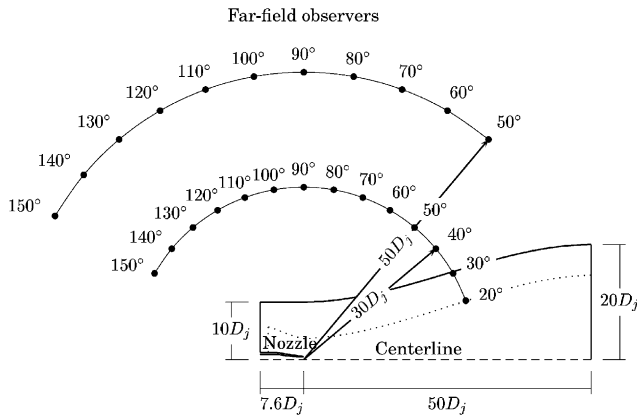


Fig. 1. Kirchhoff surface (dotted line) and far-field observer locations.

The strength of the sources in the hydrodynamic jet region decays slowly downstream, which means that the downstream end of a closed surface will enter regions of considerable hydrodynamic fluctuations. It is thus common practice to use Kirchhoff surfaces not closed in the upstream and downstream ends. It has been shown by Freund et al. (1996) that the errors introduced by using such surfaces are small if the main portion of the sound sources are within the axial extent of the surface and if lines connecting observer locations with locations in the hydrodynamic region, representing the main sources of sound, intersect with the surface.

In this work a Kirchhoff surface closed in the upstream end and open in the downstream end was used, see Fig. 1.

4. Numerical method

4.1. Numerical scheme

The governing equations were solved using a cell centered finite volume method solver. A low-dissipation third-order upwind scheme was used for discretization of the convective fluxes and a second-order central difference approach for the viscous fluxes. The convective scheme is a combination of centered and upwind biased components that have been used with good results for free shear flows by Mårtensson et al. (1991) and, more recently, for shock/shear-layer interaction by Wollblad et al. (2004). Appendix A gives more detail on the convective flux scheme.

Time marching is performed using a second-order three-stage Runge–Kutta technique. For more detail on the numerical scheme used, see Eriksson (1995) or Andersson (2003). The computations were performed in part on a Sun-cluster using 14 SunBlade 750 MHz processors and in part on a Linux cluster using 14 AMD 1700⁺ processors. MPI (Message Passing Inter-

face) routines were implemented for processor communication.

4.2. Computational domain

The computational domain consists of a boundary-fitted block structured mesh with 50 blocks and a total of about 3.0×10^6 cells excluding the buffer region at the domain outlet, see Figs. 2–4. The grid cells are concentrated to the shear-layer area. To establish mesh homogeneity, a combination of polar and cartesian blocks was used, see Fig. 4. The grid cells are stretched in the downstream direction and radially toward the boundaries using cubic Hermite grid point distribution. The last contraction of the JEAN project nozzle

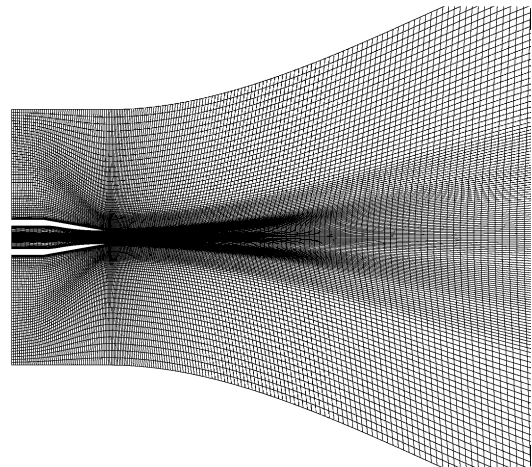


Fig. 2. A slice through the calculation domain made at $z = 0$, i.e. a xy -plane, is depicted. The figure shows the domain inlet including the nozzle and the outer boundaries in the radial direction. The axial extent of the computational domain is roughly twice that shown in the figure.

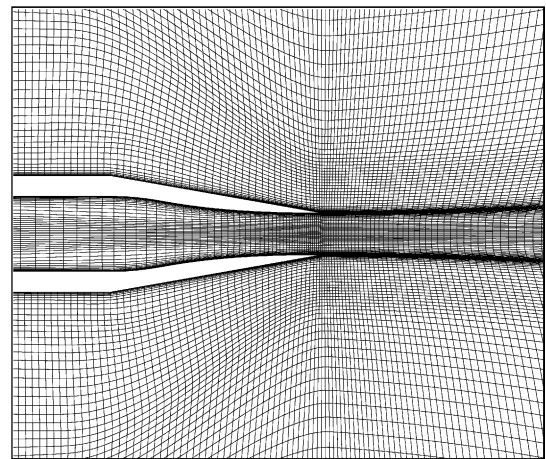


Fig. 3. A slice of the calculation domain in the nozzle region is shown. Cells are concentrated to the shear-layer area.

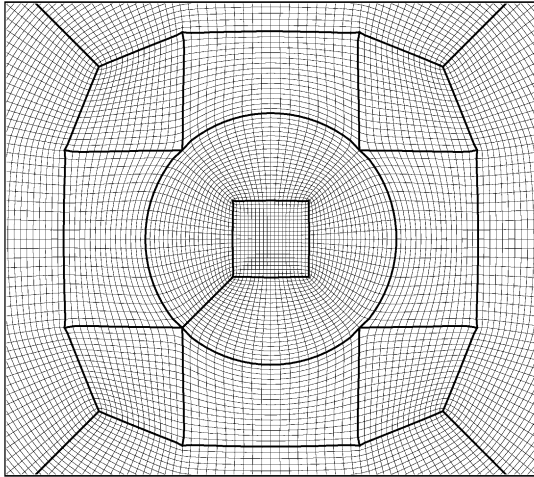


Fig. 4. A slice through the calculation domain at constant x , i.e. a yz -plane, is depicted. Combining Cartesian and polar grid blocks enhances the radial direction grid homogeneity throughout the domain.

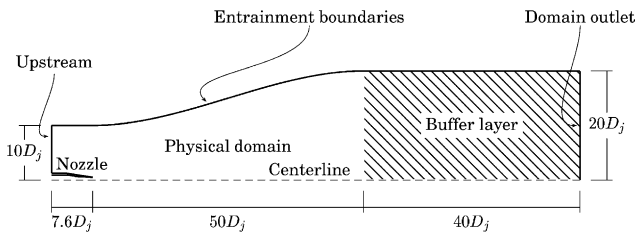


Fig. 5. Computational setup. $D_j = 50$ [mm].

is included in the calculation domain. The axial extent of the physical part of the domain is 2.5 m, which is equal to 50 nozzle diameters ($D_j = 50$ [mm]). The radial extent is 10 nozzle diameters at the nozzle exit plane and 20 diameters at the domain outlet, see Fig. 5.

4.3. Boundary conditions

At the inlet of the nozzle, the nozzle plenum, total pressure and total enthalpy are specified. All free boundaries, i.e. upstream and entrainment boundaries, are defined using absorbing boundary conditions based on characteristics variables. At the domain outlet static pressure is specified. In Fig. 5 the main boundary condition locations are depicted.

Defining proper boundary conditions is an issue of great importance, especially in aeroacoustic applications since acoustic pressure fluctuations are small and spurious waves generated at the boundaries might contaminate the acoustic field. Definitions of boundary conditions for free shear flows are particularly difficult since there are, by definition, no bounding surfaces. The difficulties in defining boundary conditions for free shear flows have been discussed in several publications

(e.g. Colonius et al., 1993; Mankbadi et al., 2000; Bogey et al., 2000b; Rembold et al., 2002).

At a finite distance downstream the jet, boundary conditions that mimic the jet behavior at infinity are to be specified. Energetic vorticity and entropy waves traveling out of the domain reaching the outlet boundary will, if not damped out, generate strong acoustic waves traveling back into the domain. These acoustic reflections can be diminished by in some way decreasing the amplitude of fluctuations of vorticity and entropy waves approaching the boundary. Inward traveling acoustic waves may still be generated but will be weaker than for a boundary where no special treatment of the boundary region is utilized. Moreover, these weaker acoustic waves are additionally damped when traveling through the outlet region on their way back into the calculation domain and, hopefully, the waves are so weak when reaching the jet region that they do not affect the results. In order to dampen acoustic reflections, an extra outlet zone has been added to the calculation domain, Fig. 5, where a damping term defined by

$$\varepsilon(Q - \langle Q \rangle) \quad (13)$$

has been added to the equation system, ε is defined by a constant, $\varepsilon_{\max}$ and the axial location, x , in the damping zone as

$$\varepsilon = \varepsilon_{\max} \left(\frac{x - x_{\min}}{x_{\max} - x_{\min}} \right)^2. \quad (14)$$

In Eq. (13), Q represents the flow variables and the time-average of Q is calculated as

$$\langle Q \rangle = \frac{\sum_{i=1}^n Q_i t_i}{\sum_{i=1}^n t_i}, \quad (15)$$

where subscript i denotes time step. The weighted time-average defined by Eq. (15) gives the recent flow property values higher weight than older values and it also gives a good estimate of the time-averaged flow field. The fluctuations of the flow field are damped in the boundary zone by forcing the flow properties toward the time-averaged flow field. Furthermore, the cells in this part of the calculation domain are more stretched than cells in the physical part of the calculation domain, which increases the numerical dissipation and thereby further damps flow field fluctuations. The boundary zone consists of roughly 5.0×10^5 cells, which is about 14% of the total calculation domain, and has an axial extent of 40 jet diameters, see Fig. 5.

The entrainment boundaries proved to be rather troublesome as well. The problem is to get enough fluid entrainment into the domain. The effect of not getting the entrained mass flow correct is that a deficit of mass is compensated with an inflow of fluid from the domain outlet. This back flow results in a recirculation zone surrounding the jet and prevents it from spreading.

Approximate values of the velocities specified at the entrainment boundaries were obtained from RANS calculations with $U_\infty = 0$ (Eriksson, 2002). These RANS calculations were performed using significantly larger calculation domain than in the LES and therefore give reliable information on the entrainment effect at the boundary of the LES domain. Furthermore, these RANS calculations have been extensively validated against experimental data (Jordan et al., 2002a). The fact that the boundary values at the entrainment boundaries have been obtained from RANS results does, to a certain degree, determine the time-averaged flow field. However, since this work is not an attempt to prove that large-eddy simulation predicts the flow of a jet properly but rather to obtain a database of flow properties for jets and to get input for calculations of the acoustic field, this is no severe disadvantage.

5. Results

Profiles of statistical quantities shown in Figs. 6–14 are obtained from flow field data that were averaged in both time and the azimuthal direction to establish

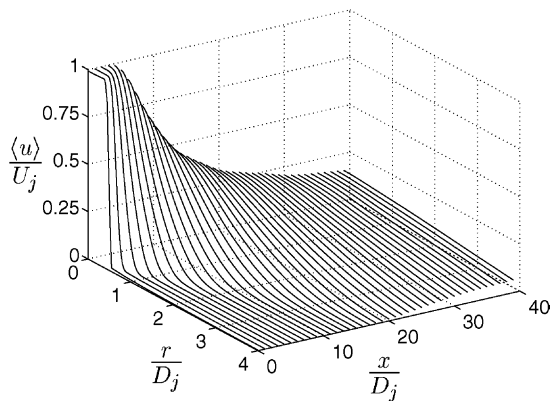


Fig. 6. Downstream development of radial profiles of axial velocity.

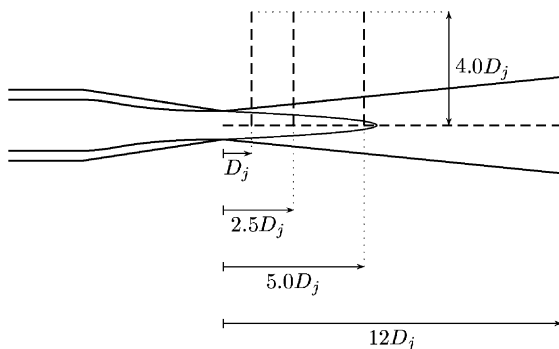


Fig. 7. Profiles of axial velocity and second-order moments were obtained along the centerline and along three radial lines.

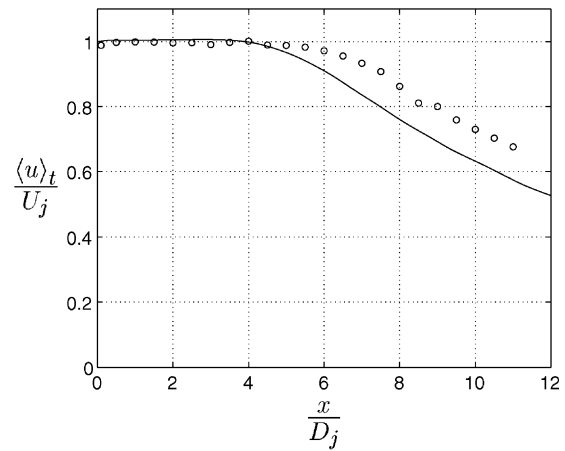


Fig. 8. Center line velocity: the solid line, ‘—’, corresponds to LES results and the circles, ‘O’, to experimental data (Jordan et al., 2002a).

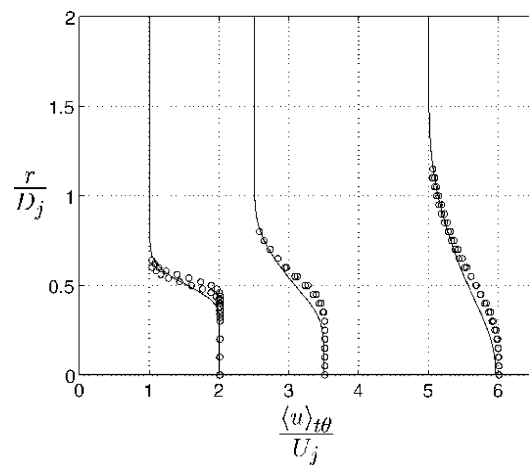


Fig. 9. Radial profiles of axial velocity at the axial positions $x/D_j = 1.0, x/D_j = 2.5$ and $x/D_j = 5.0$: ‘—’ corresponds to LES results and ‘O’ to experimental data (Jordan et al., 2002a). The profiles have been staggered corresponding to their axial position.

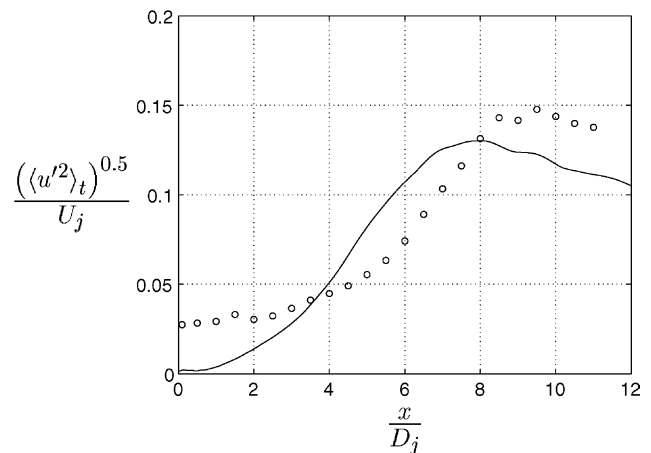


Fig. 10. Axial profile of $(\langle u'u' \rangle_t)^{0.5}$: ‘—’ corresponds to LES results and ‘O’ to experimental data (Jordan et al., 2002a).

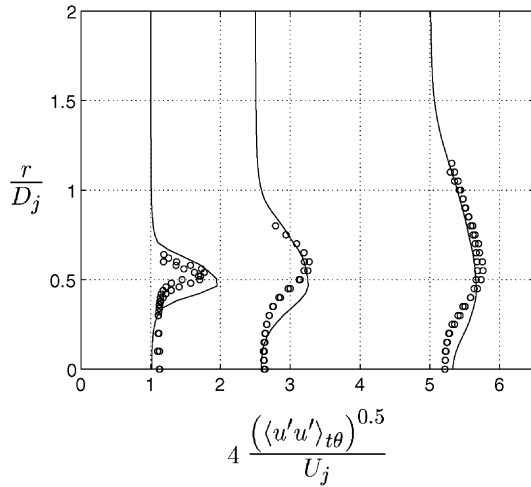


Fig. 11. Radial profiles of $(\langle u'u' \rangle_{t\theta})^{0.5}$. See also legend to Fig. 9.

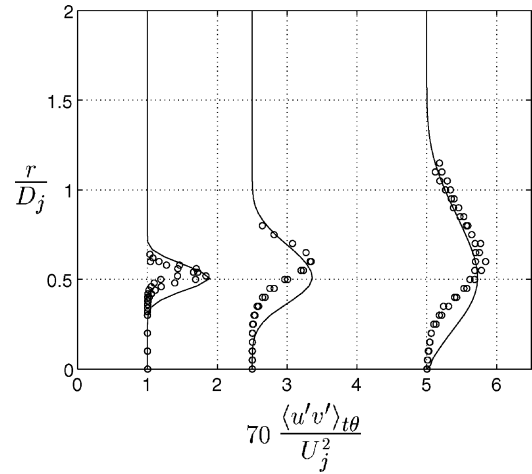


Fig. 14. Radial profiles of $(u'v')_{t\theta}$. See also legend to Fig. 9.

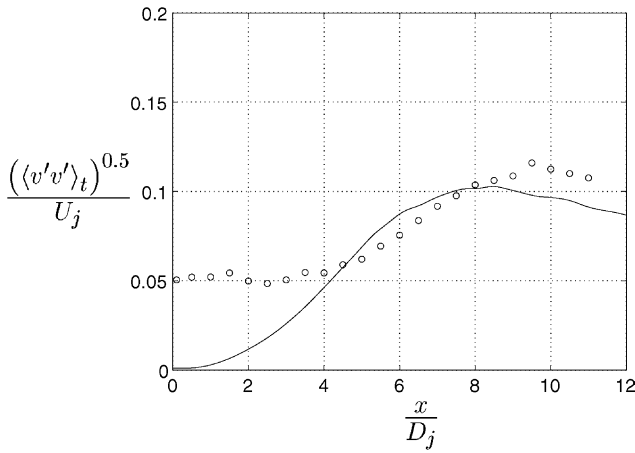


Fig. 12. Axial profile of $(\langle v'v' \rangle_t)^{0.5}$; ‘—’ corresponds to LES results and ‘○’ to experimental data (Jordan et al., 2002a).

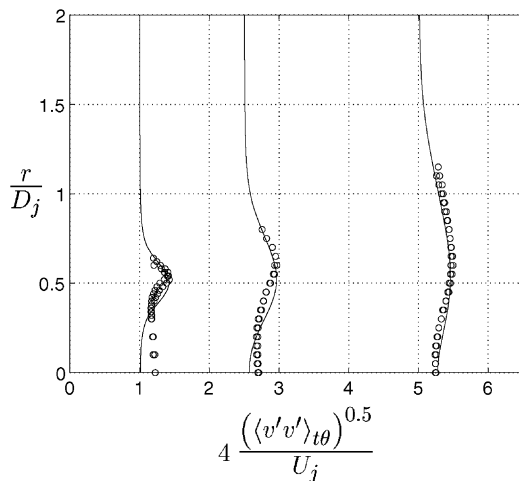


Fig. 13. Radial profiles of $(\langle v'v' \rangle_{t\theta})^{0.5}$. See also legend to Fig. 9.

improved statistical convergence. The profiles are made non-dimensional using the jet velocity at the nozzle exit, U_j , and the nozzle outlet diameter, D_j .

The calculations were started up using an initial solution interpolated from RANS. The simulation ran for about 1.0×10^5 time steps before sampling was initiated. This corresponds to 0.025 s of simulated time using a time step of 0.25 [μs] or roughly 3.5 acoustic through-flows, i.e. the time required for an acoustic wave to travel through the calculation domain, not including the outlet buffer layer, based on the speed of sound at ambient conditions. Sampling of statistic data was then continued for another 20 acoustic through-flows. Each acoustic through-flow corresponds to roughly 32 h of computer wall-time using 14 AMD 1700⁺ processors of our Linux cluster. The time step is chosen such that the CFL (Courant–Friedrichs–Lewy) number was not larger than 0.5 any where.

5.1. Time-averaged profiles

Fig. 6 depicts the development of radial profiles of axial velocity downstream of the nozzle exit. Comparison of profiles obtained from LES and experiments are presented in Figs. 8–14. The radial profiles have been staggered corresponding to the axial position, i.e. 1.0, 2.5 and 5.0 jet diameters respectively, to show downstream development. Furthermore, it should be noted that, in order to obtain similar scales for all the radial profile figures, root-mean-square values and $u'v'$ correlations have been multiplied by a factor 4 and 70, respectively. The location of the lines along which axial and radial profiles have been extracted is shown in Fig. 7.

Although the LES results are generally in good agreement with experiments, there are some deviations. The main discrepancy is that the LES fails to predict the length of the jet potential core, see Fig. 8, and thereby the location of maximum turbulence intensity, see Figs.

10 and 12. It has been mentioned by DeBonis and Scott (2002) that this is generally not the case in earlier studies of jets; rather, the length of the potential core is usually overpredicted. Although the location of the potential core closure is not accurately predicted, the rate of decay of the centerline axial velocity is.

Centerline profiles of u'_{rms} and v'_{rms} are shown in Figs. 10 and 12, respectively. The maximum level is in both cases somewhat underpredicted but, as can be seen by comparing the two figures, the turbulence anisotropy of the jet has been correctly predicted. Significant deviations from experimental data in root-mean-square values of radial velocity near the centerline, Figs. 12 and 13, might be due to low frequency flutter of the potential core found in the experiments (Jordan et al., 2002a) but not present in the computations. Compared to the measured profiles, the peaks of the predicted u'_{rms} are shifted toward the centerline for the radial profiles at $x/D_j = 1.0$ and $x/D_j = 2.5$, see Fig. 11. This is consistent with the shift of the radial profiles of axial velocity, i.e. the maximum turbulence intensity should appear at a radial location close to the maximum radial derivative of the axial velocity. However, the peak levels of u'_{rms} are very close to those of the measured profiles, which is surprising since the radial gradients is smaller for the predicted axial velocity. The overpredicted $u'v'$ correlation, see Fig. 14, is an indication of the turbulent mixing being too high and thus totally in line with the shorter potential core. For a more correct comparison, the subgrid scale component of the shear stress, $u''v''$, should be added to the resolved, $u'v'$. However, as can be seen in Fig. 15 where the sub-grid scale shear stress is compared with the resolved shear stress, the subgrid scale component is as most roughly 4.0% of the resolved and can thus be neglected.

The reason for the underprediction of the length of the potential core region can probably be found in a combination of several factors. The most important are probably subgrid scale viscosity, numerical scheme, entrainment boundary conditions and inflow conditions. DeBonis and Scott (2002) reported an underprediction of the potential core region when using LES for a supersonic jet. It was discussed in this work that the reason for this underprediction was lack of subgrid dissipation. Furthermore, it has been mentioned by Morris et al. (2000) that a change in the subgrid scale model constants affects the potential core length and the initial growth of the jet. DeBonis (2004) found that the potential core region could be extended by increasing the subgrid scale dissipation. This is a quite natural effect since the increased subgrid scale viscosity decreases the turbulent fluctuations and hence the shear-layer mixing becomes less effective. This in turn leads to an elongated region of potential flow. Figs. 16 and 17 depicts subgrid scale viscosity over molecular viscosity and subgrid scale kinetic energy over resolved turbulent kinetic energy, respectively. The profiles of subgrid scale viscosity indicate that the LES is of good quality. From Fig. 17 it can be seen that the subgrid scale kinetic energy is as most about 1.0% of the resolved. Lack of subgrid scale dissipation might be a reason for the underpredicted potential core seen in the present study. It is, however, not obvious that this is the only reason for this shorter potential core; rather it might be one of several factors. Tucker (2004) used a scheme for the convective fluxes based on a weighted average of centered and upwind biased components. It was found that even with a weak blending of fifth order upwinding with a fourth order central scheme, i.e. only small contributions from the upwind biased components, the effects of the upwind scheme is still evident. However, the effect of the artificial

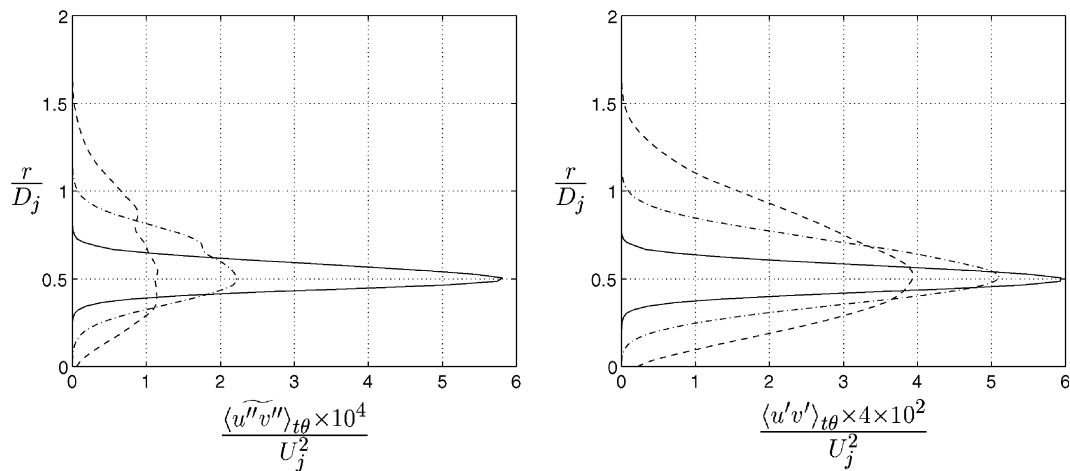


Fig. 15. Radial profiles of subgrid scale (left) and resolved (right) $u'v'$ correlations. It should be noted that the values in the left figure have been multiplied by a factor 10^4 and those in the right figure by a factor 4×10^2 ; the solid line denotes $x/D_j = 1.0$, dash-dotted line $x/D_j = 2.5$ and dashed line $x/D_j = 5.0$.

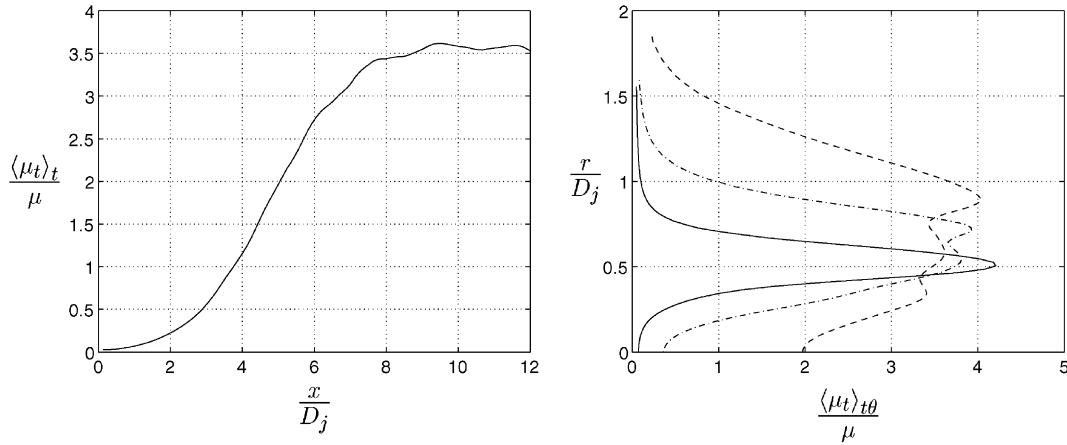


Fig. 16. Axial (left) and radial (right) profiles of subgrid scale viscosity over molecular viscosity. In the right figure, the solid line denotes $x/D_j = 1.0$, dash-dotted line $x/D_j = 2.5$ and dashed line $x/D_j = 5.0$.

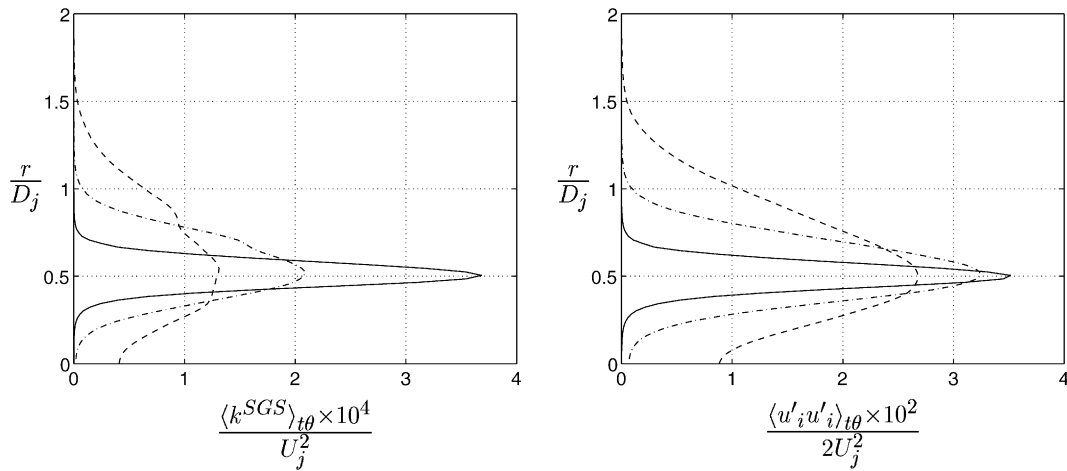


Fig. 17. Radial profiles of subgrid scale (left) and resolved (right) turbulence kinetic energy. Note that to be able to compare the profiles, the subgrid values have been multiplied by a factor 10^4 and the resolved values by a factor 10^2 . See also legend to Fig. 16.

dissipation introduced by the numerical scheme used ought to be the same as that of the subgrid scale dissipation, i.e. more numerical dissipation leads to a longer potential core due to less efficient turbulent mixing. Consequently, the numerical scheme used for the present work does not introduce too much artificial dissipation since the turbulent mixing is overpredicted. Concerning the effects of the entrainment flow, only the correct mass flow will give the correct flow behavior in the hydrodynamic region. As discussed in Section 4.3, lack of entrained mass will be compensated by back flow, which results in a recirculation zone surrounding the jet and prevent it from spreading correctly. However, the RANS computations from which the boundary entrainment velocities have been interpolated, gives a somewhat overpredicted potential core length. Other effects that may well be important are the velocity profile and turbulence level at the nozzle exit. Having a velocity profile at the nozzle exit that is not fully developed re-

sults in steep velocity gradients and therefore a more violent encounter with the irrotational surrounding fluid. This may result in larger fluctuations and thus also higher degree of turbulent mixing. [Bogey and Bailly \(2003\)](#) investigated the effects of the inflow conditions on the jet flow. It was found that the use of a thinner shear-layer momentum thickness resulted in higher turbulence intensities in the shear-layer and hence a more rapid transition. In summary, the correct potential core length may be obtained by modification of the subgrid scale properties alone. However, a more physical approach would be to modify the inflow conditions to produce results in better agreement with measurements.

5.2. Self-preservation

Radial profiles of axial velocity and second-order moments were normalized by centerline velocity and plotted versus a non-dimensional radial coordinate to

investigate whether or not self-preservation has been established. These profiles were obtained using statistical flow field data averaged in time and the azimuthal direction. The constant, x_0 , used to collapse the profiles is a virtual origin obtained from the least square fit of data points in Fig. 18, i.e. the axial position for which $U_j/\langle u_c \rangle_t = 0$. In this case $x_0 = 0.11D_j$. As seen in Fig. 18, a non-negligible deviation from the linear relation occurs for centerline points located between $20D_j$ and $35D_j$, downstream of the nozzle exit. This behavior is probably due to the entrainment boundary conditions used. However, since this occurs far downstream, the effect on the flow field in the region near the nozzle exit ($x < 20D_j$) is probably negligible. Fig. 19 shows collapsed radial profiles of axial velocity. Five radial profiles equally spaced between $19D_j$ and $39D_j$ were used. The jet spreading rate can be estimated to be roughly

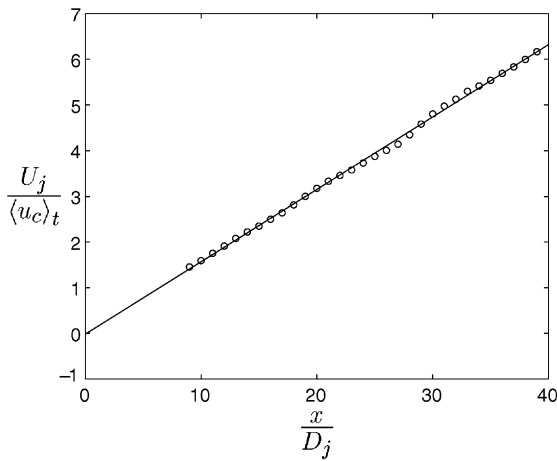


Fig. 18. Predicted center line velocity. The solid line corresponds to a curve fit to the LES data. The x -value for which the line intersects the x -axis, x_0 , is used to collapse the radial profiles in Figs. 19–23.

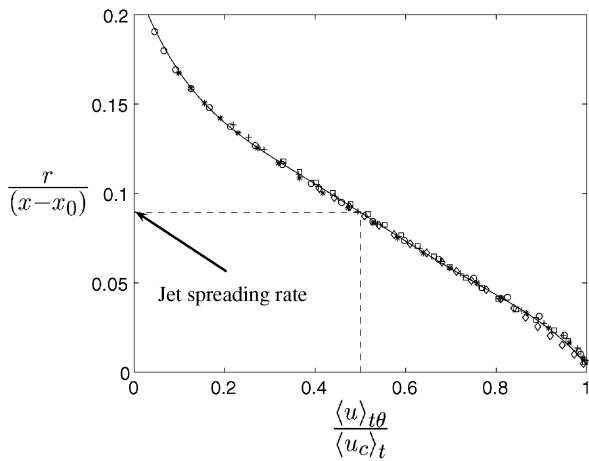


Fig. 19. Radial profiles of axial velocity normalized by centerline mean velocity. The solid line corresponds to a curve fit to the LES data. The profiles are labeled as follows: '○' = $19D_j$, '*' = $24D_j$, '+' = $29D_j$, '□' = $34D_j$ and '◇' = $39D_j$.

0.09 based on the curve fitted to the LES data. Figs. 20–23 depict normalized profiles of second-order moments. For these figures, radial profiles in six axial positions from $29D_j$ to $39D_j$ downstream of the nozzle were used. The distance between each profile is, in this case, two jet diameters. As can be seen in the Figs. 19–23, self-preservation of axial velocity has been well established within 39 jet diameters downstream of the nozzle whereas, for self-preservation of second-order moments to be obtained data would have to be extracted further downstream. For example Hussein et al. (1994) found that profiles of second and higher order moments displayed self-similar behavior when evaluated at axial positions more than $70D_j$ downstream of the virtual

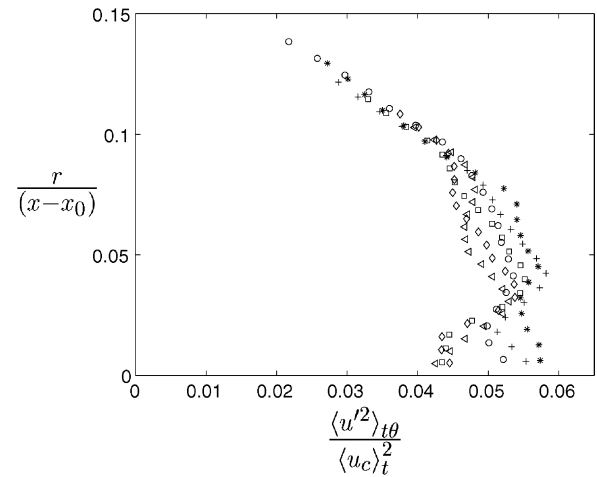


Fig. 20. Radial profiles of $\langle u'u' \rangle_{t\theta}$ normalized by centerline mean velocity. Radial profiles at six axial locations are represented. The profiles are labeled as follows: '○' = $29D_j$, '*' = $31D_j$, '+' = $33D_j$, '□' = $35D_j$, '◇' = $37D_j$ and finally '◁' = $39D_j$.

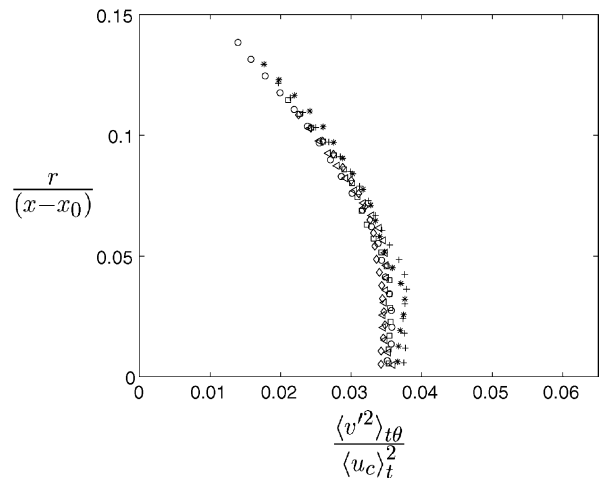


Fig. 21. Radial profiles of $\langle u'v' \rangle_{t\theta}$ normalized by centerline mean velocity. For legend see Fig. 20.

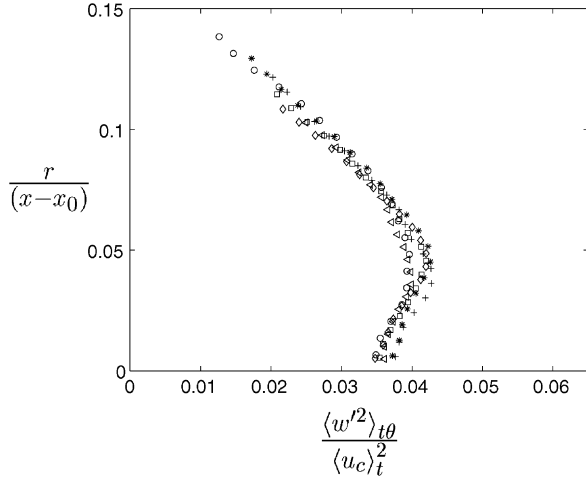


Fig. 22. Radial profiles of $\langle w'^2 \rangle_{t\theta}$ normalized by centerline mean velocity. For legend see Fig. 20.

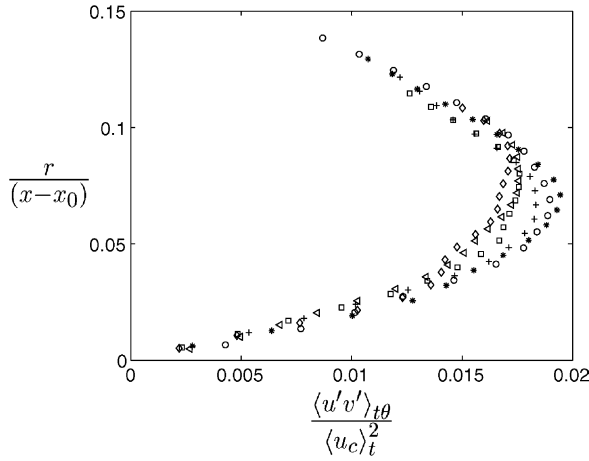


Fig. 23. Radial profiles of $\langle u'v' \rangle_{t\theta}$ normalized by centerline mean velocity. For legend see Fig. 20.

origin. However, this would require a larger computational domain to be used.

5.3. Two-point space-time correlations

Two-point space-time correlations were obtained for axial velocity in a few points in the shear-layer region downstream of the nozzle exit. The time series used is 0.0725 s. Spatial separations, ξ , are in this case downstream in the axial direction. The two-point space-time correlation of axial velocity for a certain spatial separation, ξ , and separation in time, τ , is given by

$$\mathcal{R}_{uu}(\mathbf{x}, \xi, \tau) = \frac{\langle u'(\mathbf{x}, t)u'(\mathbf{x} + \xi, t + \tau) \rangle_t}{(\langle u'^2(\mathbf{x}) \rangle_t)^{0.5}(\langle u'^2(\mathbf{x} + \xi) \rangle_t)^{0.5}}, \quad (16)$$

where u' denotes fluctuation of axial velocity and $\mathbf{x}$ is the position in the flow field where the two-point correlation is evaluated, see Fig. 24. Figs. 25–27 show two-point



Fig. 24. Two-point space-time correlations in the shear-layer are obtained for location, $\mathbf{x}$, using a spatial separation, ξ .

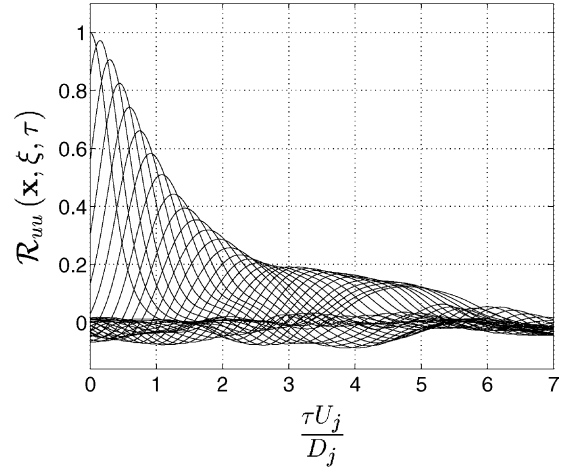


Fig. 25. Two-point space-time correlation of axial velocity in the shear-layer at $r = 0.5D_j$, $x = 2.5D_j$. Each curve corresponds to a two-point space-time correlation obtained using a certain spatial separation, ξ . Starting with no separation in space, i.e. the autocorrelation, the spatial separation is increased by $0.1D_j$ per curve.

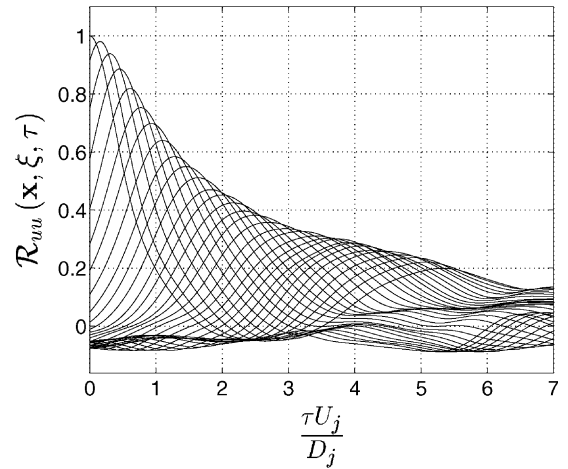


Fig. 26. Two-point space-time correlation of axial velocity in the shear-layer at $r = 0.5D_j$, $x = 5.0D_j$. See also legend to Fig. 25.

correlations in points located on the nozzle lip-line 2.5, 5.0 and 10.0 nozzle diameters downstream of the nozzle exit. Each figure includes the autocorrelation of axial velocity fluctuations, i.e. no spatial separation, and 30 two-point space-time correlations obtained with increased spatial separation. Each consecutive correlation curve corresponds to an increase in spatial separation by $0.1D_j$. The integral of the envelope built up of the two-point space-time correlation curves corresponds to a

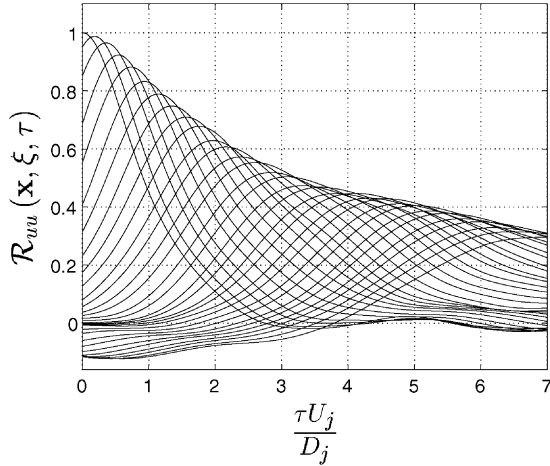


Fig. 27. Two-point space-time correlation of axial velocity in the shear-layer at $r = 0.5D_j$, $x = 10.0D_j$. See also legend to Fig. 25.

convective time scale in the current position in the flow. Comparing Figs. 25–27, the amplitude for a given spatial separation, ξ , increases downstream the jet, which indicates that the turbulence becomes more and more frozen in character, i.e. the turbulence length scale increases as the jet develops. A comparison was made with two-point measurements in the shear-layer at the end of the potential core, $x = L_c$. Here the potential core length is defined as the axial location where the centerline mean velocity equals $0.95U_j$. For the LES, this gives $L_c = 5.45D_j$ and, for the measured data, $L_c^{\text{exp}} = 6.50D_j$. For more detail on the two-point measurements see Jordan and Gervais (2003). In Fig. 28, correlation peak locations of the two-point space-time correlations obtained from LES data are compared with those from the two-point measurements. The agreement is, as can

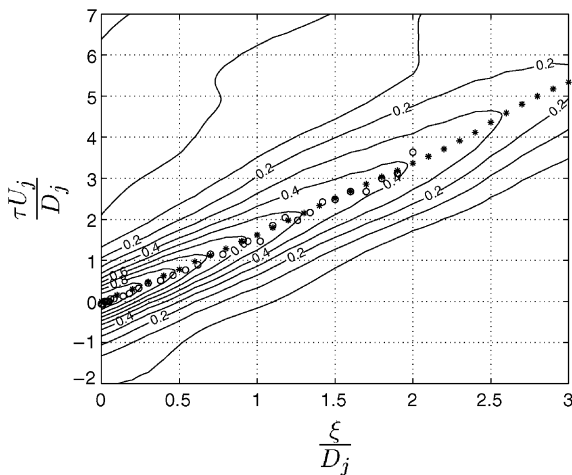


Fig. 28. Contours of $\mathcal{R}_{uu}(\mathbf{x}, \xi, \tau)$ obtained in the shear-layer, $r = 0.5D_j$, at the end of the potential core, $x = L_c$. The contours corresponds to LES data, '*' corresponds to the correlation peaks of these two-point space-time correlations and 'O' to the peaks of the correlations at $x = L_c^{\text{exp}}$ obtained from the velocity data measured by Jordan and Gervais (2003).

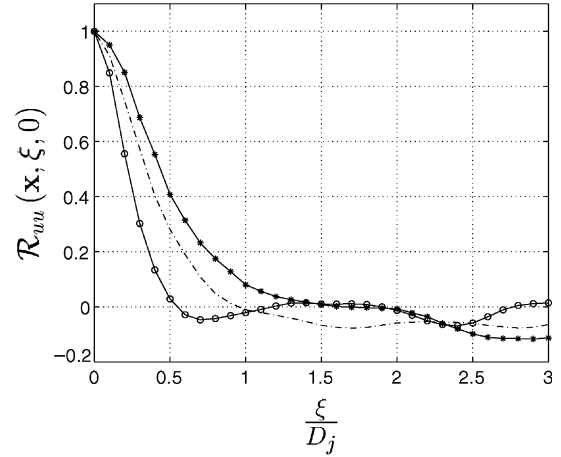


Fig. 29. Predicted two-point correlation of axial velocity at three different locations in the shear-layer: '—O—' corresponds to a location $x = 2.5D_j$ downstream the nozzle exit, '---' to $x = 5.0D_j$ and '—*' to $x = 10.0D_j$. All points are located at $r = 0.5D_j$.

be seen, good. Fig. 29 shows predicted spatial correlations of axial velocity in three shear-layer locations. Here it can again be seen that the turbulence length scale increases downstream.

5.4. Turbulence length and time scales

Length and time scales of turbulence quantities have been obtained using the two-point space-time correlations presented in the previous section. Estimates of convection velocity of turbulence structures were also obtained for a number of axial locations in the shear-layer. The separation in time between the peak of the autocorrelation curve and the peak of the first following two-point space-time correlation curve was used. This time delay corresponds to a spatial separation of $0.1D_j$, as mentioned previously, and thus gives an estimate of the convection velocity, see Fig. 30. A more correct way would be not to use the temporal separation for the peak but rather the time for which the two-point space-time correlation envelope is a tangent to the correlation curve (Fisher and Davies, 1963). However, the result would not differ a great deal. Predicted convection velocities for a few axial locations are presented in Table 2. In the middle row, the convection velocities have been normalized by the jet velocity, U_j , and, in the right row, by the local mean velocity. As shown in Table 2, the eddy convection velocity is everywhere higher than the local mean velocity. This might be a nonintuitive result but is a phenomenon observed in experiments (Fisher and Davies, 1963; Wygnanski and Fiedler, 1969) and is caused by skewness of the velocity distribution. In shear-layer locations close to the high velocity potential core, the convection velocity is likely to be larger than the local mean velocity and lower in the outer parts of the shear-layer (Fisher and Davies, 1963).

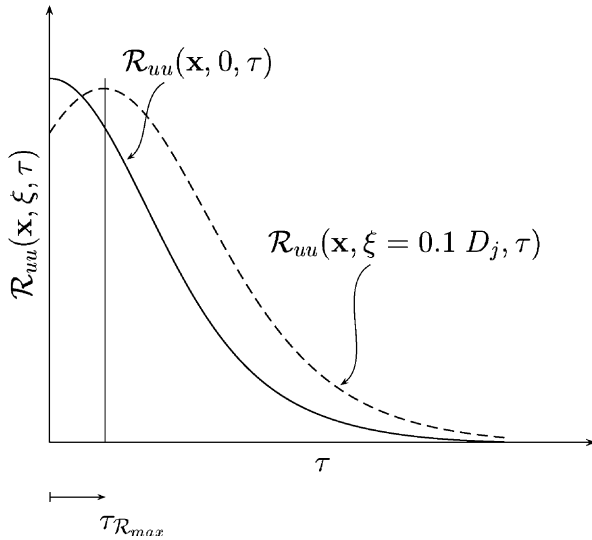


Fig. 30. The convection velocity of turbulence structures was estimated using the temporal separation, $\tau_{R_{max}}$.

Local integral length and time-scales are estimated by integration of the corresponding autocorrelation and spatial correlation, respectively. The first crossing of the coordinate axis is used as the upper limit for the integration. The axial development of integral length and time scales are shown in Figs. 31 and 32. In these figures, scales obtained from measured data are represented by circles. The agreement between predictions and experiments is good.

5.5. Far-field sound pressure levels

Comparisons of predicted far-field sound pressure levels (SPL) and levels measured by Jordan et al. (2002b) are shown in Figs. 33 and 34. The time series of far-field pressure fluctuations used for the evaluation contains about 1.5×10^4 samples and is roughly 0.038 s. The observer locations for which sound pressure levels are presented in Fig. 33 are located on a $30D_j$ radius arc and those in Fig. 34 on a $50D_j$ radius arc. The observer locations in relation to the Kirchhoff surface used are shown in Fig. 1. The sound pressure level is here defined as

$$\text{SPL} = 20 \log_{10} \left(\frac{(\langle (p')^2 \rangle_t)^{0.5}}{p_{\text{ref}}} \right), \quad (17)$$

where

$$p_{\text{ref}} = (\rho_{\infty} c_{\infty} 10^{-12})^{0.5} = 2.0 \times 10^{-5} \text{ [Pa]}, \quad (18)$$

Table 2

Predicted convection velocity of turbulence structures in the shear-layer, $r = 0.5D_j$

x/D_j	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00
u' / U_j	0.71	0.71	0.65	0.60	0.71	0.65	0.65	0.56
$u' / \langle u \rangle_t$	1.19	1.23	1.15	1.09	1.31	1.24	1.28	1.16

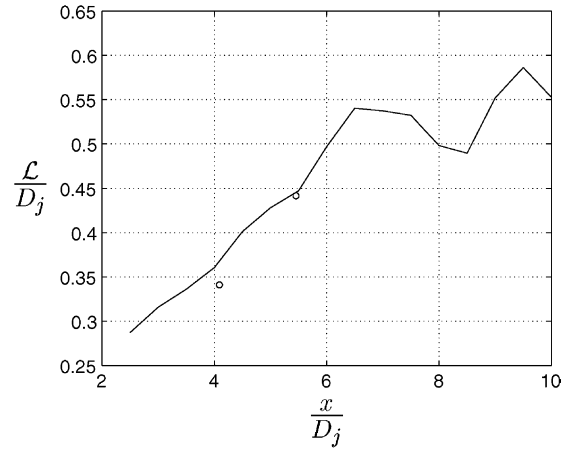


Fig. 31. Axial development of integral length scale in the shear-layer, i.e. $r = 0.5D_j$. 'O' denotes experiments (Jordan and Gervais, 2003).

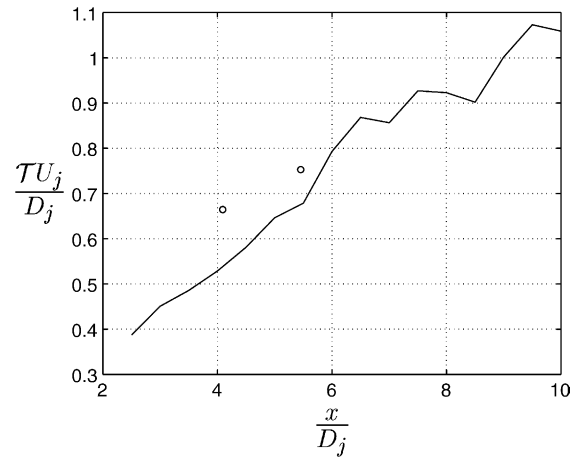


Fig. 32. Axial development of integral time scale in the shear-layer, i.e. $r = 0.5D_j$. 'O' denotes experiments (Jordan and Gervais, 2003).

corresponds to the threshold of human hearing at a frequency of 1000 Hz. Sound pressure levels obtained using all spectral information available are often referred to as overall sound pressure levels (OASPL). The time history of pressure in the observer locations was obtained numerically using instantaneous flow data from the LES in combination with Kirchhoff surface integration. As can be seen in the figures, predicted levels agree very well with experimental data. The predicted sound pressure levels are for all observer locations within a 3 dB deviation from the measured levels and for most observer locations within a 1 dB deviation. The predicted lev-

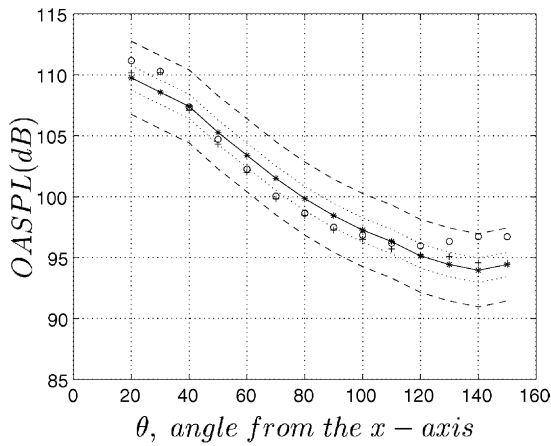


Fig. 33. Far-field sound pressure levels in microphone locations $30D_j$ from the nozzle exit: ‘*’ corresponds to sound pressure levels measured by Jordan et al. (2002b), the dashed lines to the measured levels ± 3 dB and the dotted lines to the measured levels ± 1 dB. ‘○’ denotes predicted sound pressure levels using Kirchhoff surface integration and ‘+’ SPL obtained using bandpass filtered pressure signals.

els are not as good for angles below 40° and higher than 120° as for those in between. This is especially noticeable for the lower arc. In this case the fact that the observer locations are very close to the Kirchhoff surface might affect the levels obtained. Furthermore, removing the high frequency contribution and the contribution from very low frequencies by band-pass filtering the observer pressure signals significantly improves the results, see

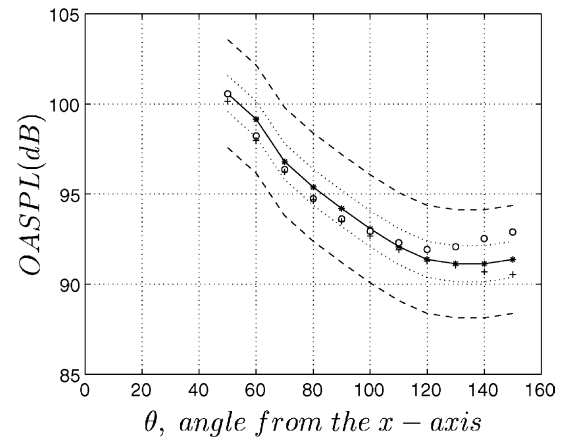


Fig. 34. Far-field sound pressure levels in microphone locations $50D_j$ from the nozzle exit. See also legend to Fig. 33.

Figs. 33 and 34. The lower and upper cut-off frequencies of the band-pass filter used corresponds to $St = 0.05$ and $St = 1.0$, respectively. Fig. 35 shows a comparison of the power spectrum of predicted pressure fluctuations in a couple of observer locations with corresponding spectra obtained for the measured data (Jordan et al., 2002b). For low and high angles the predicted levels are in good agreement with the experimental data up to at least Strouhal number $St = 1.5$, see Fig. 35(a). For the intermediate angles on the other hand, the predicted amplitude decreases rapidly above $St = 1.0$. This is due to the fact that the grid is not fine enough to support high

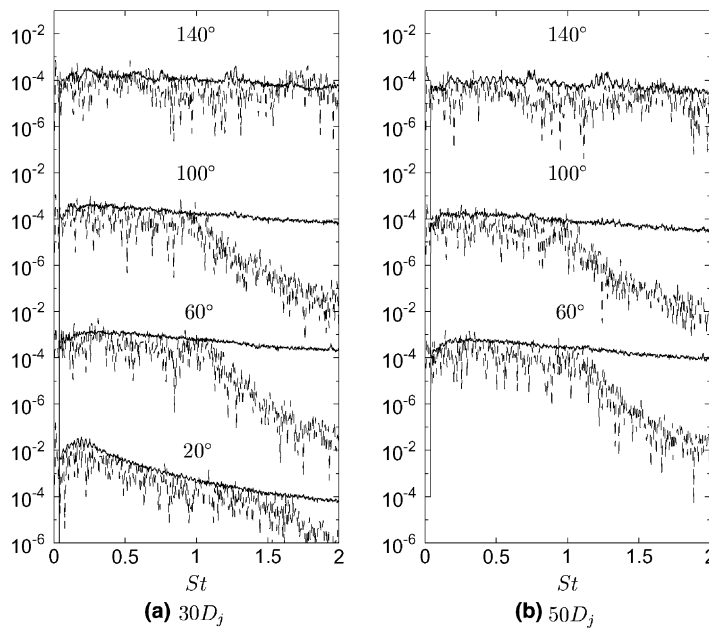


Fig. 35. Power spectrum of far-field pressure fluctuations versus Strouhal number, $St = fD_j/U_j$. The dashed line, ‘- -’ corresponds to predicted data obtained using Kirchhoff integration and the solid line corresponds to experimental data measured by Jordan et al. (2002b). In figure (a), four observer locations are represented. Starting at a location 20° from the x -axis, the angle is increased by an increment of 40° per spectrum. In figure (b) the first spectrum from below is obtained for an observer location 60° from the x -axis, i.e. it should be compared with the second spectra from below in figure (a). Note that the spectra have been staggered and the scale on the y -axis therefore is repeated.

frequency acoustic waves in the radial direction. Assuming that four cells per wavelength are required to be able to capture a propagating wave of a certain frequency, the cells in the area where the Kirchhoff surface is located would support propagating waves of Strouhal numbers up to $St \simeq 1.2$.

6. Concluding remarks

An LES of a compressible Mach 0.75 nozzle/jet configuration has been performed. The Reynolds number based on jet velocity and nozzle diameter was 5.0×10^4 . Profiles of time and azimuthally averaged flow properties were compared with data measured by [Jordan et al. \(2002a\)](#). Although some deviations occur, the results are generally in good agreement with experiments. The good results are probably attributed to the homogeneity of the mesh used and the fact that a nozzle geometry has been included in the calculation domain. Using a combination of Cartesian and polar mesh blocks makes it possible to ensure that cells in the outer regions of the calculation domain are not too large and prevent cells from being clustered in the centerline region. The initial jet spreading and the potential core length are, however, not predicted correctly. Overpredicted $u'v'$ correlation close to the nozzle exit indicates that the degree of mixing is too high which makes the transition process too effective and hence the length of the potential region becomes shorter. This may be due to a combination of several factors such as: lack of subgrid scale dissipation, the numerical method used, entrainment boundary conditions and inflow conditions. Of these, the subgrid scale dissipation and the inflow conditions are believed to be the most important. Increasing the amount of subgrid scale dissipation would elongate the potential core region due to less effective mixing. Moreover, the initial mixing could probably be decreased by modification of the inflow conditions.

Two-point space-time correlations were obtained in a number of locations in the shear-layer along the nozzle lip-line. Correlation curves obtained at $x = L_c$ were in good agreement with experiments ([Jordan and Gervais, 2003](#)). Two-point space-time correlations in the shear-layer were used to obtain estimates of integral length scales, integral time scales and eddy convection velocities. The agreement with corresponding quantities obtained from the measured data was satisfying.

Sound pressure levels in the far-field were evaluated with a hybrid approach. Kirchhoff surface integration was utilized for propagation of sound to far-field locations. Instantaneous pressure on the Kirchhoff surface was obtained from the LES. Predicted sound pressure levels are in excellent agreement with measured levels ([Jordan et al., 2002b](#)). However, the mesh used fails to

capture waves of Strouhal numbers higher than $St \simeq 1.2$. This results in a rapid fall-off of the spectrum amplitudes for higher Strouhal numbers. The lower part of the spectra is accurately captured, however. Since the sound pressure levels obtained were in good agreement with the measured levels, capturing the lower part of the spectra seems to be sufficient to represent the main part of the radiated sound.

Acknowledgment

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Appendix A. Convective flux scheme

The coefficients of low-dissipation upwind scheme used for estimation the convective flux over a cell face are derived using a third-order polynomial $Q(x)$ to represent the variation of the flow state in the direction normal to the face:

$$Q(x) = A + Bx + Cx^2 + Dx^3. \quad (\text{A.1})$$

Assuming that the grid is equidistant, the cell averages of Q , represented by $\bar{Q}_1$ to $\bar{Q}_4$ in [Fig. A.1](#), can be obtained by integrating $Q(x)$ over the cells as

$$\begin{aligned} \int_{-2}^{-1} Q(x) dx &= A - \frac{3}{2}B + \frac{7}{3}C - \frac{15}{3}D = \bar{Q}_1, \\ \int_{-1}^0 Q(x) dx &= A - \frac{1}{2}B + \frac{1}{3}C - \frac{1}{4}D = \bar{Q}_2, \\ \int_0^1 Q(x) dx &= A + \frac{1}{2}B + \frac{1}{3}C + \frac{1}{4}D = \bar{Q}_3, \\ \int_1^2 Q(x) dx &= A + \frac{3}{2}B + \frac{7}{3}C + \frac{15}{4}D = \bar{Q}_4. \end{aligned} \quad (\text{A.2})$$

By rearranging the expressions in [\(A.2\)](#), the coefficients A , B , C and D can be expressed as functions of the state vectors $\bar{Q}_1$ to $\bar{Q}_4$:

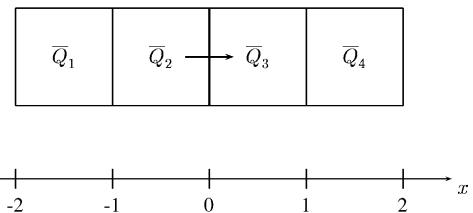


Fig. A.1. The face state is estimated using the state in four neighboring cells.

$$\begin{aligned}
A &= \frac{1}{12} [-\bar{Q}_1 + 7\bar{Q}_2 + 7\bar{Q}_3 - \bar{Q}_4], \\
B &= \frac{1}{12} [\bar{Q}_1 - 15\bar{Q}_2 + 15\bar{Q}_3 - \bar{Q}_4], \\
C &= \frac{1}{4} [\bar{Q}_1 - \bar{Q}_2 - \bar{Q}_3 + \bar{Q}_4], \\
D &= \frac{1}{6} [-\bar{Q}_1 + 3\bar{Q}_2 - 3\bar{Q}_3 + \bar{Q}_4].
\end{aligned} \tag{A.3}$$

The face state is evaluated using the interpolated value $Q(0)$, modified to include upwinding by adding the third derivative of $Q(x)$ according to

$$\begin{aligned}
Q_0 &= Q(0) + \delta Q'''(0) = A + 6\delta D \\
&= C_1\bar{Q}_1 + C_2\bar{Q}_2 + C_3\bar{Q}_3 + C_4\bar{Q}_4,
\end{aligned} \tag{A.4}$$

where the coefficient, δ , in front of the upwind term has been chosen by numerical experiment (Mårtensson et al., 1991) to be $\frac{1}{96}$ in order to only introduce a small amount of upwinding. The result is the low-dissipation third-order upwind scheme used in this work:

$$\begin{cases} C_1 = -(\frac{1}{12} + \delta) = -\frac{9}{96}, \\ C_2 = (\frac{7}{12} + 3\delta) = \frac{59}{96}, \\ C_3 = (\frac{7}{12} - 3\delta) = \frac{53}{96}, \\ C_4 = (\frac{1}{12} - \delta) = -\frac{7}{96}. \end{cases} \tag{A.5}$$

A left hand side and right hand side version of Q_0 are obtained as

$$\begin{aligned}
Q_L &= C_1\bar{Q}_1 + C_2\bar{Q}_2 + C_3\bar{Q}_3 + C_4\bar{Q}_4, \\
Q_R &= C_1\bar{Q}_4 + C_2\bar{Q}_3 + C_3\bar{Q}_2 + C_4\bar{Q}_1,
\end{aligned} \tag{A.6}$$

where subscripts L and R denotes left and right, respectively. The characteristic variables on the cell face are obtained using the estimates of the face state (A.6). Upwinded face states are then obtained using the characteristic variables. Whether the left hand side version or the right hand side version of Q should be used for calculation of the characteristic variables is determined by the signs of the characteristic speeds. The characteristic speeds are in turn obtained using a face state based on the average of $\bar{Q}_2$ and $\bar{Q}_3$.

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Paper II

Large-Eddy Simulation of Subsonic Turbulent Jets and Their Radiated Sound

By

N. Andersson, L.-E. Eriksson and L. Davidson

AIAA Journal, 43(9), pp. 1899–1912, 2005.

Large-Eddy Simulation of Subsonic Turbulent Jets and Their Radiated Sound

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Large-eddy simulations of a compressible nozzle/jet configuration have been carried out. Two jets were simulated, an isothermal jet and a jet with a higher temperature than the quiescent surrounding air. The Mach number was in both cases 0.75, and the jet Reynolds number was 5.0×10^4 . Sound pressure levels in far-field observer locations were evaluated using Kirchhoff surface integration. The Favre filtered Navier–Stokes equations were solved using a finite volume method solver with a low-dissipation third-order upwind scheme for the convective fluxes, a second-order centered difference approach for the viscous fluxes, and a three-stage second-order Runge–Kutta technique in time. The computational domain was discretized using a block structured boundary-fitted mesh with approximately 3.0×10^6 cells. The calculations were performed on a parallel computer, using message-passing interface. A compressible form of Smagorinsky’s subgrid-scale model was used for computation of the subgrid-scale stresses. Absorbing boundary conditions based on characteristic variables were adopted for all free boundaries. Velocity components specified at the entrainment boundaries were estimated from corresponding Reynolds-averaged Navier–Stokes calculations, which enable the use of a rather narrow domain. This, furthermore, ensures that the correct amount of fluid is entrained into the domain. Two-point space–time correlations were obtained for locations in the shear layer center, from which length and timescales of turbulence structures were evaluated. Predicted near-field flow statistics and far-field sound pressure levels (SPL) are both in good agreement with experiments. Predicted SPL are for all observers locations, where evaluated, within a 3.0-dB deviation from measured levels and for most locations within a 1.0-dB deviation. Experimental data used for validation were provided by Laboratoire d’Etude Aérodynamiques, Poitiers, France.

Nomenclature

C_p	=	specific heat at constant pressure
C_R, C_I	=	Smagorinsky model coefficients
c	=	speed of sound
D_j	=	nozzle outlet diameter
e	=	energy
f	=	frequency
k	=	kinetic energy
$\mathcal{L}$	=	integral length scale
L_c	=	potential core length
M	=	Mach number
Pr	=	Prandtl number
p	=	pressure
Q	=	state vector
q_j	=	energy diffusion vector
$\mathcal{R}$	=	correlation amplitude
r	=	radial coordinate or distance from source to observer
Re_D	=	Reynolds number based on the jet diameter
S_{ij}	=	strain rate tensor
St	=	Strouhal number, $(f D_j)/U_j$
T	=	temperature
$\mathcal{T}$	=	integral timescale
t	=	time
(u, v, w)	=	axial, radial, and tangential velocity component
u_i	=	Cartesian components of velocity vector
$\mathbf{x}$	=	flowfield location

x_i	=	Cartesian coordinate vector component
$\mathbf{y}$	=	observer location
Δ	=	filter width
δ_{ij}	=	Kronecker delta
θ	=	angle from the x axis
μ	=	dynamic viscosity
ξ	=	spatial separation
ρ	=	density
σ_{ij}	=	viscous stress tensor
τ	=	temporal separation
τ_{ij}	=	subgrid-scale stress tensor
τ_r	=	retarded time
$\langle \dots \rangle_\theta$	=	circumferentially averaged quantity
$\langle \dots \rangle_t$	=	time-averaged quantity

Subscripts

j	=	jet, nozzle-exit condition
t	=	turbulent quantity
0	=	total condition
∞	=	freestream or ambient conditions

Superscripts

SGS	=	subgrid scale
—	=	spatially filtered quantity
$\prime$	=	resolved fluctuation
$\sim$	=	spatially Favre-filtered quantity
$\wedge$	=	Fourier transform

I. Introduction

THE number of commercial aircraft in service is continuously increasing, and airports around the world are growing in size. Moreover, new airports are often built in the vicinity of cities. Restrictions of noise levels in the surroundings of airports have made the reduction of near-ground operation noise an important issue for aircraft and engine manufacturers, and noise generation has now become an important design parameter that is taken into consideration early in the construction process. Flow-induced aircraft noise can

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be divided into two categories: airframe noise and noise generated by the jet engine. The first category includes noise generated by landing gear, high-lift devices, and the aircraft fuselage itself, and the second includes turbomachinery noise, core noise, and jet noise. At takeoff, the main sources of noise are the propelling jet and the engine fan, of which the jet exhaust is usually the strongest noise source at full power. In commercial aircraft, increasing the bypass ratio, that is, the ratio of air passing through the engine in the bypass duct to the air passing through the engine core, has given a significant reduction in aircraft noise. However, although increasing the bypass ratio leads to reduced noise levels, the major motivation for this development has not been noise reduction. Rather, the development towards more efficient engines has led to the use of a higher bypass ratio with noise reduction as a positive side effect. The lower noise levels of high-bypass ratio engines are directly attributable to the reduction in jet noise resulting from lower jet velocities. Unfortunately, without a step change in technology the maximum bypass ratio is limited by a number of factors, for example, the length of the fan blades, rotor speed, and engine nacelle drag, and large engines are currently very close to this limit. Consequently, the possibility for reducing jet noise by increasing the bypass ratio is rapidly decreasing.

New noise-reducing concepts will arise from a better understanding of the source mechanisms. For evaluation of the performance of these new concepts, reliable methods for modeling of the source mechanisms must be available. The work presented in this paper has been done within a European Union (EU) project, JEAN (Jet Exhaust Aerodynamics and Noise, EU 5th Framework Project, Contract G4RD-CT2000-000313), that focused on investigating jet noise both numerically and experimentally. Various known methodologies for noise prediction were tested in the project and compared for a few test cases.

Using a grid fine enough in the far-field regions to minimize the introduction of sound propagation errors, the acoustic field can be obtained directly from the flowfield simulation. This requires a detailed numerical compressible flow simulation, for example, direct numerical simulation (DNS), for example, see Freund¹ and Mitchell et al.² or large-eddy simulation (LES), as done by, for example, Bogey et al.³ and Mankbadi et al.⁴ With the computational resources available today, DNS is restricted to flows with fairly low Reynolds number. Moreover, it is believed⁵ that large scales are more efficient than small ones in generating sound, which justifies the use of LES for sound predictions. Although it is possible to obtain the acoustic signature directly from the flow simulation, resolving the entire acoustic field is computationally expensive. To lower the computational costs, a hybrid approach can be used in which the computational problem is divided into two parts. An LES can be used to obtain the unsteady nonlinear near field, which in the jet-noise case corresponds to the hydrodynamic jet region. The acoustic field is then extended to far-field observer locations using, for example, Kirchhoff surface integration (see Freund et al.⁶).

The feasibility of using LES for both the flowfield and the radiated sound from a high subsonic 6.5×10^4 Reynolds number jet has been discussed by Bogey et al.^{3,7,8} In Refs. 3 and 7, the acoustic field was obtained directly from the flow simulation. Noise-generation mechanisms were found to be relatively independent of Reynolds number. In Ref. 8, Lighthill's⁹ acoustic analogy was used in combination with compressible LES to obtain the acoustic field. Bogey and Bailly¹⁰ investigated the effects of inflow conditions on the flowfield and the radiated sound of a high-Reynolds-number, $Re_D = 4.0 \times 10^5$, Mach 0.9 jet. Both the flow development and the emitted sound were shown to depend appreciably on initial parameters. Bodony and Lele¹¹ investigated the changes in jet mean field and far-field sound with temperature and Mach number. Results were found to be in agreement with published data.¹² Freund¹ investigated sources of sound in a Mach 0.9 jet at a Reynolds number of $Re_D = 3.6 \times 10^3$ using DNS. In this work the part of the Lighthill source that can radiate to the far field was isolated using Fourier methods. It was found that the peak of the radiating source coincides with neither the peak of the total source nor the peak of turbulence kinetic energy. Shur et al.¹³ made simulations of a cold Mach 0.9 jet at a Reynolds num-

ber of 1.0×10^4 . Radiated sound was successfully predicted using the Ffowcs Williams–Hawkings¹⁴ surface integral formulation. The simulation was conducted using only 5.0×10^5 cells. This work was done using the monotone-integrated LES approach, where the subgrid-scale model, essentially, is replaced by the numerical dissipation of the numerical method used. Zhao et al.¹⁵ conducted an LES of a Mach 0.9 jet at Reynolds number 3.6×10^3 and a jet at Mach 0.4 and a Reynolds number of 5.0×10^3 . In this study, radiated sound was obtained both directly from the LES and by using Kirchhoff surface integration. The effect on the radiated sound of the subgrid-scale model used was investigated. It was found that using a mixed subgrid-scale model resulted in both higher turbulence levels and sound levels.

II. Overview of the Present Study

Large-eddy simulations of a Mach 0.75 nozzle/jet configuration were performed. The Reynolds number based on the nozzle-exit diameter and the jet velocity at the nozzle-exit plane Re_D was 5.0×10^4 . The Reynolds number in the measurements by Jordan et al.^{16,17} and Jordan and Gervais,¹⁸ used for comparison and validation of the simulations, was approximately 7.5×10^5 . Such a high Reynolds number probably means that the scales that need to be resolved are too small. Thus the Reynolds number in our LES was decreased with the assumption that the flow is only weakly Reynolds number dependent. This Reynolds-number reduction will affect the turbulence in the initial region and thus the frequency content of the radiated sound. However, the differences will probably be most noticeable in the high-frequency regime and thus not very important here because these high frequencies are to a high degree filtered by the numerics and therefore not included in the analysis.

Simulations of the Mach 0.75 jet were performed for two flow conditions: an unheated jet, that is, a jet where the static temperature in the nozzle-exit plane T_j was equal to the static temperature of the ambient air T_∞ and a heated version in which the exhaust temperature was twice that of the surrounding air. The main reason for choosing the Mach 0.75 jet was that experimental data were available for both isothermal and heated conditions. This made it possible to investigate whether heating effects on both the flowfield and on the radiated sound could be accurately captured with the LES/Kirchhoff approach. Furthermore, both the jet Mach number and the temperature ratio used for the heated jet are quite realistic for real jet engines. Table 1 gives flow properties defining the flowfields of the two jets simulated. Differences are highlighted using boldface numbers. The two jets, that is, the isothermal and heated Mach 0.75 jet, will hereafter be referred to as jet 1 and jet 2, respectively.

The nozzle geometry used for the simulations corresponds to the last contraction of the nozzle configuration in the experimental setup.

The measurements were made by Jordan et al.^{16,17} and Jordan and Gervais¹⁸ at the MARTEL facility of Centre d'Etudes Aérodynamiques et Thermiques, Poitiers, France. The measurements have also been reported by Power et al.¹⁹ Two-component single-point and monocomponent two-point measurements were made using laser Doppler velocimetry. The acoustic field was sampled using an arc of microphones at 30 jet diameters and 50 jet diameters from the jet exit, respectively. For more detail on the experimental setup, see Jordan et al.^{16,17} and Jordan and Gervais.¹⁸

Table 1 Flow properties

Parameter	Jet 1	Jet 2
U_j/c_∞	0.75	0.75
T_j/T_∞	1.0	2.0
P_∞ , Pa	101,300	101,300
ρ_∞ , kgm ⁻³	1.225561	1.225561
c_∞ , ms ⁻¹	340.174	340.174
U_∞ , ms ⁻¹	0.0	0.0
T_∞ , K	288.0	288.0
T_{0j} , K	320.4	608.4
Re_D^a	5.0×10^4	5.0×10^4

^aThe Reynolds number was decreased using modified viscosity.

III. Governing Equations

The equations solved are the spatially Favre-filtered continuity, momentum, and energy equations,

$$\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial (\bar{\rho} \tilde{u}_i)}{\partial x_i} = 0 \quad (1)$$

$$\frac{\partial (\bar{\rho} \tilde{u}_i)}{\partial t} + \frac{\partial (\bar{\rho} \tilde{u}_i \tilde{u}_j)}{\partial x_j} = -\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial \bar{\sigma}_{ij}}{\partial x_j} + \frac{\partial \tau_{ij}}{\partial x_j} \quad (2)$$

$$\begin{aligned} \frac{\partial (\bar{\rho} \tilde{e}_0)}{\partial t} + \frac{\partial (\bar{\rho} \tilde{e}_0 \tilde{u}_j)}{\partial x_j} = & -\frac{\partial \bar{p} \tilde{u}_j}{\partial x_j} \\ & + \frac{\partial}{\partial x_j} \left(C_p \frac{\mu}{Pr} \frac{\partial \tilde{T}}{\partial x_j} + q_j \right) + \frac{\partial}{\partial x_j} [\tilde{u}_i (\bar{\sigma}_{ij} + \tau_{ij})] \end{aligned} \quad (3)$$

where $\bar{\sigma}_{ij}$ and τ_{ij} are the Favre-filtered viscous stress tensor and subgrid-scale viscous stress tensor, respectively. These are here defined as

$$\bar{\sigma}_{ij} = \mu \left(2\tilde{S}_{ij} - \frac{2}{3}\tilde{S}_{mn}\delta_{ij} \right) \quad (4)$$

$$\tau_{ij} = \mu_t \left(2\tilde{S}_{ij} - \frac{2}{3}\tilde{S}_{mn}\delta_{ij} \right) - \frac{2}{3}\bar{\rho} k^{SGS} \delta_{ij} \quad (5)$$

where k^{SGS} is the subgrid-scale kinetic energy

$$k^{SGS} = C_I \Delta^2 \tilde{S}_{mn} \tilde{S}_{mn} \quad (6)$$

μ_t is the subgrid-scale dynamic viscosity

$$\mu_t = C_R \bar{\rho} \Delta^2 \sqrt{\tilde{S}_{mn} \tilde{S}_{mn}} \quad (7)$$

and $\tilde{S}_{ij}$ is the Favre-filtered strain rate tensor given by

$$\tilde{S}_{ij} = \frac{1}{2} \left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i} \right) \quad (8)$$

The subgrid heat flux appearing in the Favre-filtered energy equation is modeled using a temperature gradient approach

$$q_j = C_p \frac{\mu_t}{Pr_t} \frac{\partial \tilde{T}}{\partial x_j} \quad (9)$$

The filter width used in Eqs. (6) and (7) is the local grid cell width, that is, $\Delta = (\Delta_1 \Delta_2 \Delta_3)^{1/3}$. The subgrid-scale model used in this work is the Smagorinsky part of the model proposed by Erlebacher et al.²⁰ for compressible flows. The constants C_R and C_I appearing

in Eqs. (6) and (7) are the Smagorinsky model constants. These are here given by

$$C_R = 0.012 \quad C_I = 0.0066 \quad (10)$$

The system of governing equations, Eqs. (1–3), is closed by making assumptions concerning the thermodynamics of the gas considered. It is assumed that the gas is thermally perfect, that is, it obeys the gas law. Furthermore, the gas is assumed to be calorically perfect, which implies that internal energy and enthalpy are linear functions of temperature.

IV. Sound Propagation

Kirchhoff integration is a method for predicting the value of a property Φ , governed by the wave equation, at a point outside a surface enclosing all generating structures.²¹ The method was originally used in the theory of diffraction of light and in other problems of an electromagnetic nature but has recently been used extensively for aeroacoustic applications. The integral relation is given by

$$\Phi(\mathbf{y}, t) = \frac{1}{4\pi} \int_S \left[\frac{\Phi}{r^2} \frac{\partial r}{\partial n} - \frac{1}{r} \frac{\partial \Phi}{\partial n} + \frac{1}{c_\infty r} \frac{\partial r}{\partial n} \frac{\partial \Phi}{\partial t} \right] dS(\mathbf{x}) \quad (11)$$

where $\mathbf{y}$ is a observer location in the far field and $\mathbf{x}$ a location on the surface. The τ_r denotes that the expression within brackets is to be evaluated at retarded time, that is, emission time, and τ_r is related to the observer evaluation time t by the distance from the surface to the observer, $r = |\mathbf{y} - \mathbf{x}|$, and the speed of sound in the far-field region c_∞ as

$$\tau_r = t - r/c_\infty \quad (12)$$

The variable Φ to be evaluated is in this case the surface pressure. S denotes the surface enclosing all sound-generating structures and n denotes the surface normal, positive outward. The surface S must be placed in a region where the flow is completely governed by a homogeneous linear wave equation with constant coefficients.⁶ More detail on the Kirchhoff surface integration method can be found in, for example, Freund et al.⁶ and Lyrintzis.²¹

The strength of the sources in the hydrodynamic jet region decays slowly downstream, which means that the downstream end of a closed surface will enter regions of considerable hydrodynamic fluctuations (see Fig. 1). It is thus common practice to use Kirchhoff surfaces that are not closed in the upstream and downstream ends. It was shown by Freund et al.⁶ that the errors introduced by using such surfaces are small if the main portion of the sound sources are within the axial extent of the surface and if lines connecting observer locations with locations in the hydrodynamic region, representing

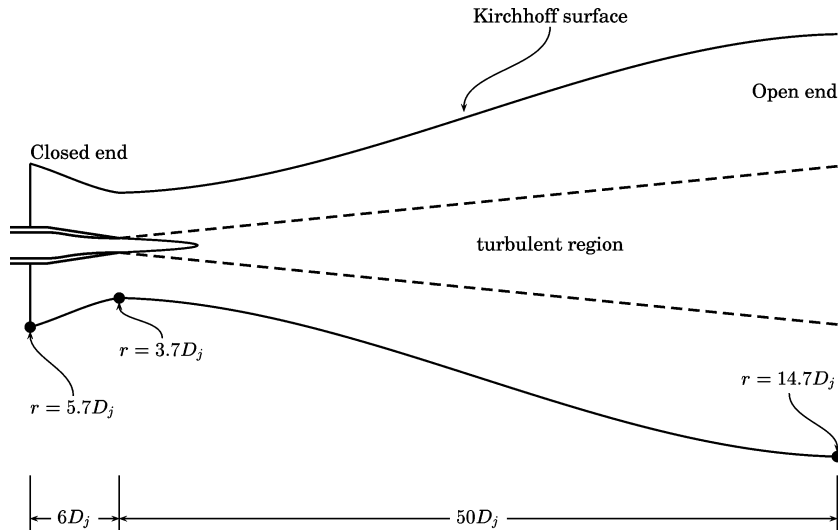


Fig. 1 Jet-fitted Kirchhoff surface closed in the upstream end and open in the downstream end.

the main sources of sound, intersect with the surface. Rahier et al.²² found that the downstream closing surface only gives a minor contribution to the radiated sound. In the work presented in this paper, a Kirchhoff surface closed in the upstream end and open in the downstream end was used (see Fig. 1).

One of the main strengths of the Kirchhoff surface integral approach, contrary to many other methods, is the fact that as long as all sources of sound, flow inhomogeneities, and objects that in some way affect the radiated sound are bounded by the surface, all of these effects are included. This, for example, means that no extra treatment has to be considered to be able to predict the effects of refraction and convection.

V. Method

A. Numerical Scheme

The Favre-filtered Navier–Stokes equations were solved using a finite volume method solver with a third-order low-dissipation upwind scheme for the convective fluxes and a centered difference approach for the diffusive fluxes. The temporal derivatives were estimated using a second-order three-stage Runge–Kutta technique. The solver used is based on the G3D family of codes developed by Eriksson.²³ The convective scheme is a combination of centered and upwind-biased components that have been used with good results for free shear flows by Mårtensson et al.²⁴ and, more recently, for shock/shear-layer interaction by Wollblad et al.²⁵ The coefficients of the low-dissipation upwind scheme used for estimating the convective flux over a cell face are derived using a third-order polynomial $Q(x)$ to represent the variation of the flow state in the direction normal to the face (see Fig. 2).

$$Q(x) = A + Bx + Cx^2 + Dx^3 \quad (13)$$

The face state is evaluated using the interpolated value $Q(0)$, modified to include upwinding by adding the third derivative of $Q(x)$ according to

$$\begin{aligned} Q_0 &= Q(0) + \delta \frac{\partial^3 Q}{\partial x^3}(0) = A + 6\delta D \\ &= C_1 \bar{Q}_1 + C_2 \bar{Q}_2 + C_3 \bar{Q}_3 + C_4 \bar{Q}_4 \end{aligned} \quad (14)$$

where the coefficient δ in front of the upwind term has been chosen by numerical experiment²⁴ to be $1/96$ to introduce only a small amount of upwinding. The result is the low-dissipation third-order upwind scheme used in this work, that is, low dissipation compared to a standard third-order upwind scheme, which is obtained using $\delta = 1/12$. The coefficients (C_1 – C_4) of the low-dissipation third-order upwind scheme are

$$\begin{aligned} C_1 &= -(1/12 + \delta) = -9/96 & C_2 &= (7/12 + 3\delta) = 59/96 \\ C_3 &= (7/12 - 3\delta) = 53/96 & C_4 &= -(1/12 - \delta) = -7/96 \end{aligned} \quad (15)$$

For more detail on the numerical scheme, see Eriksson.²³ One-dimensional convection can be described by the following relation:

$$\frac{\partial u}{\partial t} + c \frac{\partial u}{\partial x} = 0 \quad (16)$$

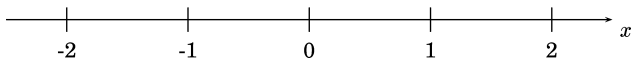
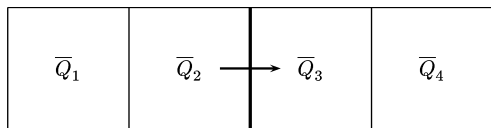


Fig. 2 Face states estimated from the cell-averaged state in four neighboring cells.

where c is a constant coefficient. Assuming an initial solution to Eq. (16) of harmonic type, its exact solution can be written as

$$u(x, t) = u_0 \exp(st) \exp(-ikx) \quad (17)$$

where u_0 is a constant amplitude and k is the wave number. The time-dependent part of the solution is $\exp(st)$ and s describes its time dependence. Inserting Eq. (17) in Eq. (16) gives $s = ikc$, which implies that the exact solution to Eq. (16) will have a periodic behavior in time without damping or amplification. Furthermore, the phase velocity will be the same for all wave numbers, namely, the constant c .

An even-order central numerical approximation to a first-order derivative does not introduce any numerical dissipation in itself. Therefore, dissipation is often added in the range of wave numbers for which the dispersion errors are significant. Artificial numerical dissipation is introduced by adding an even-order derivative, which corresponds to adding $(-1)^{n+1} \delta (\partial^{2n} u / \partial x^{2n})$ on the right-hand side of Eq. (16). Discretizing Eq. (16) using a finite volume method approach implies that first-order derivatives are estimated by differences of cell face averages. Because the cell-face-averaged properties, in the solver used in the present work, are obtained using a third-order upwind scheme where the upwinding is established by adding a third-order derivative, this corresponds to a fourth-order centered scheme with a fourth-order derivative added to introduce dissipation, in finite difference terms. The time dependence s for the discretized case, using a five-point stencil, becomes

$$\begin{aligned} s &= \frac{c}{\Delta x} \left\{ 2i \sum_{l=1}^2 a_l^{\text{FDM}} \sin(lk\Delta x) \right. \\ &\quad \left. - \delta \left[d_0^{\text{FDM}} + 2 \sum_{l=1}^2 d_l^{\text{FDM}} \cos(lk\Delta x) \right] \right\} \end{aligned} \quad (18)$$

where a_l^{FDM} corresponds to the finite difference coefficients defining the central scheme and d_l^{FDM} the finite difference coefficients defining the derivative added for upwinding. The amount of dissipation added is prescribed by the constant δ . Figure 3 shows the dispersion relation and dissipation relation of the scheme used in the present work. The dispersion relation is the imaginary part of Eq. (18), and the dissipation relation is the real part. For comparison the dispersion relation of the fourth-order dispersion-relation-preserving scheme proposed by Tam and Webb²⁶ is included in Fig. 3a. The time-stepping scheme will modify the dispersion and dissipation errors, but for the Courant–Friedrichs–Lewy (CFL) number chosen in the present study $\text{CFL} \leq 0.5$ the effects are small (see Billson²⁷).

B. Computational Setup

Figure 4 gives an overview of the computational domain used for the LES. In this figure, the upper half of the three-dimensional domain used for the calculations is shown. To minimize the effect of reflections at the domain outlet on the predicted flowfield, a damping zone was added at the domain outlet. The functionality of this damping zone or buffer layer has been described in more detail in Andersson et al.²⁸ and Andersson.²⁹ The axial extent of the physical part of the domain downstream of the nozzle exit is 2.5 m, which is equal to 50 nozzle diameters ($D_j = 50$ mm). The radial extent is $10D_j$ at the nozzle-exit plane and $20D_j$ at the domain outlet. The physical part of the computational domain was discretized using a block-structured boundary-fitted mesh with 33 mesh blocks and approximately 3×10^6 cells (see Figs. 5a and 5b). Additionally 5×10^5 cells were used to discretize the damping zone. To avoid a centerline singularity and to establish mesh homogeneity throughout the domain, a combination of polar and Cartesian blocks was used (see Fig. 5b). The grid cells are stretched in the downstream direction and radially toward the boundaries using cubic Hermite grid point distribution.

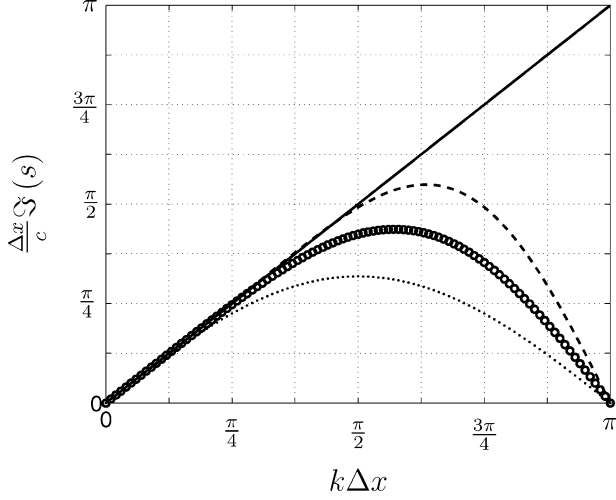
For all free boundaries, that is, the upstream and entrainment boundaries, absorbing boundary conditions based on the method of

characteristics were adopted. Static pressure was specified at the domain outlet, that is, at the downstream end of the buffer layer, and at the nozzle inlet P_0 and h_0 were specified. No unsteady forcing of incoming disturbances at the domain inlet was utilized to trigger the initial development of the jet. The velocities specified at the entrainment boundaries were interpolated from corresponding Reynolds-averaged Navier–Stokes (RANS) calculations. The calculation domain used for these RANS computations is sufficiently large to ensure that no disturbances caused by the outlet boundaries

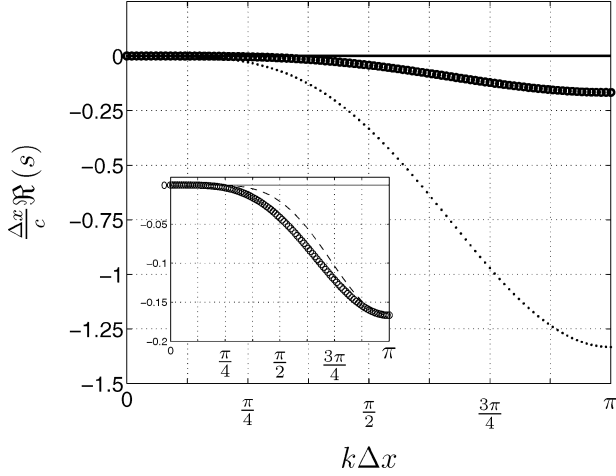
will reach the location of the LES domain boundaries. Figure 6 shows a comparison of the LES and RANS calculation domains.

VI. Results

This section gives predictions of near-field aerodynamics of a Mach 0.75 jet and its radiated sound. The calculations were started up using initial solutions interpolated from two-dimensional axisymmetric RANS calculations. For jet 1, the simulation ran for



a) Dispersion relation



b) Dissipation relation

Fig. 3 Dispersion and dissipation relations: ○, numerical scheme used in the present work; and —, denote the exact solution: a) ---, fourth-order DRP scheme proposed by Tam and Webb²⁶ and ···, dispersion relation for a second-order centered scheme; and b) ○, dissipation relation for a fourth-order derivative with an amplifying factor $\delta = 1/96$ and ···, same fourth-order derivative with $\delta = 1/12$, that is, the standard third-order upwind scheme. The enclosed figure shows a comparison of dissipation relations for a fourth-order derivative (○) and a sixth-order derivative (---).

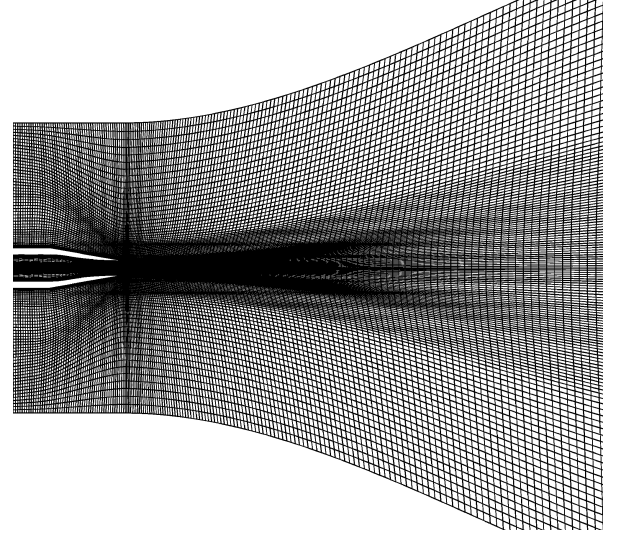


Fig. 5a Slice through the calculation domain made at $z=0$, that is, a xy plane. The figure shows the domain inlet including the nozzle and the outer boundaries in the radial direction. The axial extent of the computational domain is roughly twice that shown.

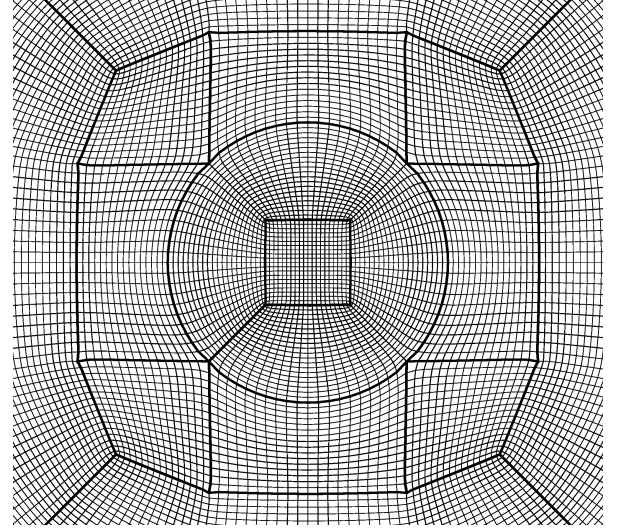


Fig. 5b Slice through the calculation domain at constant x , that is, a yz plane. Combining Cartesian and polar grid blocks enhances the radial direction grid homogeneity throughout the domain.

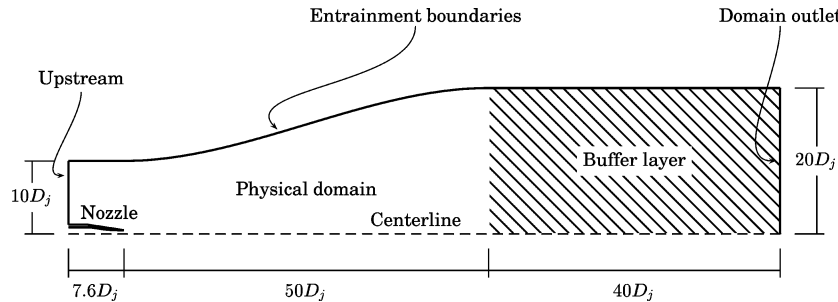


Fig. 4 Computational setup.

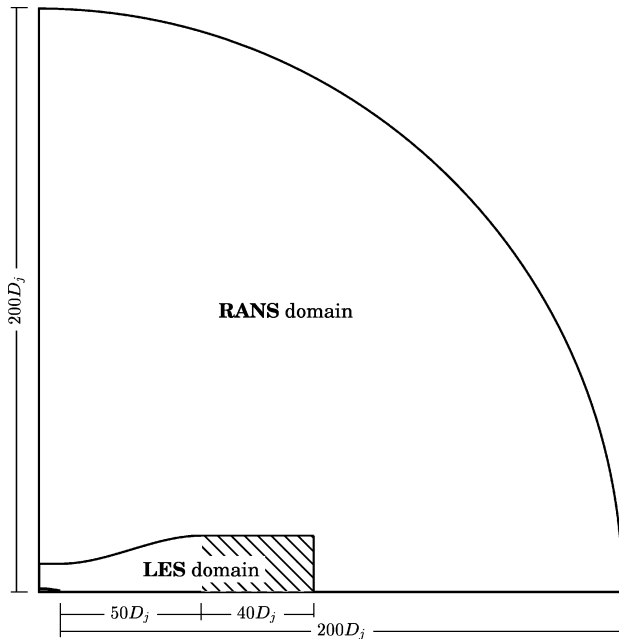


Fig. 6 Entrainment velocities were obtained from RANS calculations performed using a significantly larger domain.

about 1.0×10^5 time steps ($\Delta t = 0.25 \mu s$) before sampling was initiated. This corresponds to 0.025 s of simulated time or roughly 3.5 acoustic through-flows, that is, the time required for an acoustic wave to travel through the calculation domain, not including the outlet buffer layer, based on the speed of sound at ambient conditions. Sampling of statistical data was then continued for another 20 acoustic through-flows. Each acoustic through-flow corresponds to roughly 32 h of computer wall time using 14 AMD 1700+ processors of our Linux cluster. For jet 2, as for jet 1, about 3.5 acoustic through-flows were completed before sampling of data was initiated, which in this case corresponds to roughly 1.7×10^5 time steps ($\Delta t = 0.15 \mu s$). Sampling of statistical data was then continued for about 14 acoustic through-flows. In this case, because of the shorter time step, each acoustic through-flow corresponds to roughly 54 h of computer wall time. The time steps were chosen such that the CFL number was not larger than 0.5 anywhere. Profiles of statistical quantities to be presented in the following sections are obtained from flowfield data that were averaged in both time and the azimuthal direction to establish improved statistical convergence. The profiles are made nondimensional using the jet velocity at the nozzle exit U_j and the nozzle outlet diameter D_j . More detail on the results presented in this section can be found in Andersson²⁹ and Andersson et al.²⁸

A. Instantaneous and Time-Averaged Flowfield

Figure 7 depicts contours of instantaneous axial velocity in a plane along the jet axis, $y = 0$. Comparing the contours in Fig. 7a, which represent jet 1, with those obtained for jet 2 (Fig. 7b), it seems that heating the jet results in a higher initial mixing and hence a shorter potential core region. Furthermore, the heated jet seems to be narrower than the isothermal jet. According to Hinze,³⁰ experiments have indicated that increasing the temperature ratio T_j/T_∞ gives a narrower jet. Furthermore, it has been observed that the length of the potential core decreases with increased temperature ratio.³⁰ Figure 8 shows contours of instantaneous axial velocity in a plane cutting through the jet at $x = 3.0D_j$. These figures clearly visualize the asymmetric nature of the jet at an instant in time.

Profiles of mean-flow quantities were extracted along the centerline and along radial lines at three axial positions downstream of the nozzle exit according to Fig. 9. Although the time-averaged flowfield is generally in good agreement with the measured quantities, there are some discrepancies. In the predicted flowfield the initial mixing is overpredicted, resulting in a shorter potential core region. Figure 10 gives a comparison of predicted and measured centerline

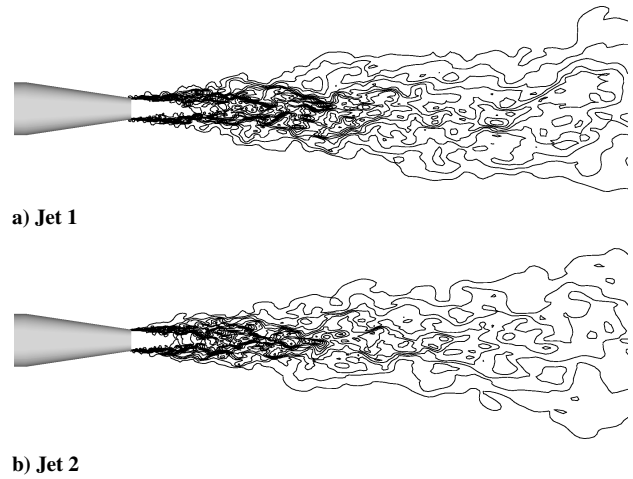


Fig. 7 Contours of instantaneous axial velocity ($y = 0$).

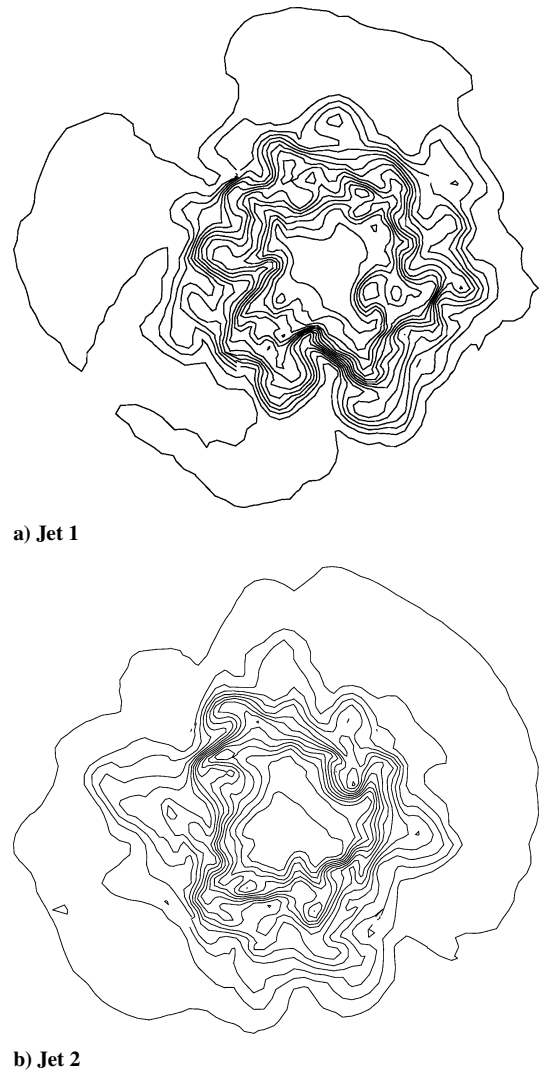


Fig. 8 Contours of instantaneous axial velocity ($x = 3.0D_j$).

axial velocity for the two jets. Defining the length of the potential core L_c as the axial location where the averaged axial velocity is $0.95 U_j$ gives for jet 1 $L_c = 5.5D_j$ and for jet 2 $L_c = 4.0D_j$, which should be compared with $L_c = 6.5D_j$ and $5.0D_j$ for the experimental data.

Figure 11 shows radial profiles of axial velocity. Figure 12 gives a comparison of turbulence intensities. The differences in levels of u'_{rms} and v'_{rms} indicate that the turbulence anisotropy is captured. The peak levels of predicted turbulence intensity are shifted toward the nozzle exit, which is consistent with the underprediction of potential

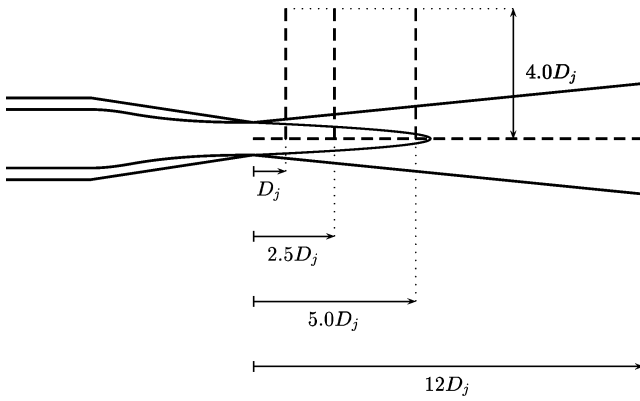
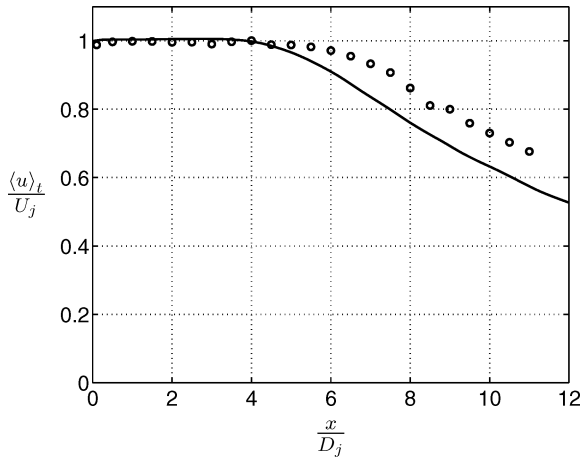
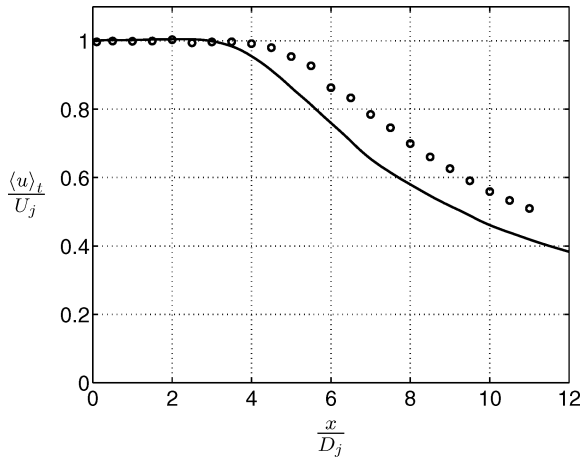


Fig. 9 Lines along which profiles of time-averaged quantities were extracted are denoted by dashed lines.



a) Jet 1

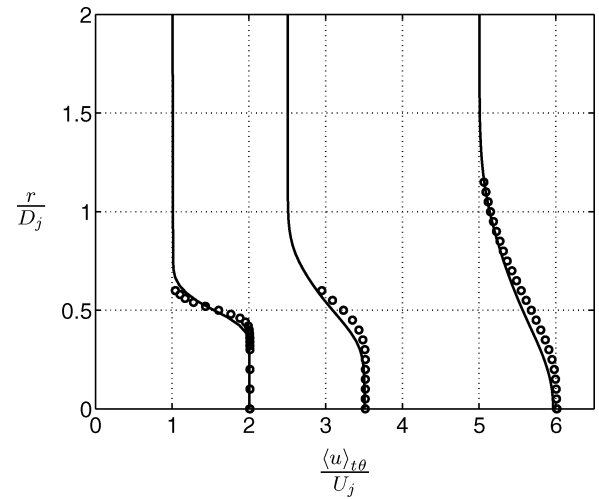


b) Jet 2

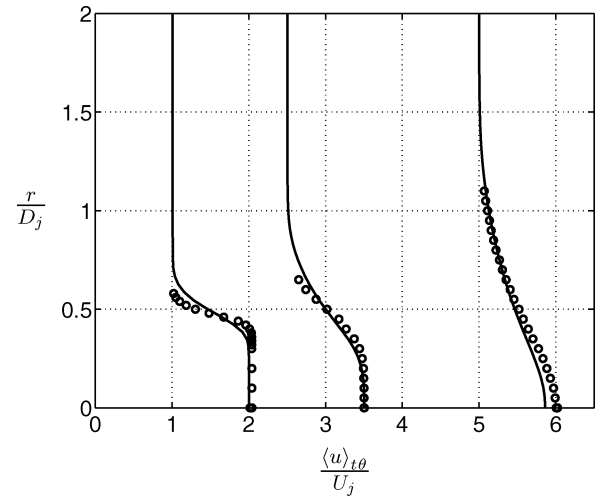
Fig. 10 Centerline profile of time-averaged axial velocity: —, LES and $\circ$, measurements.¹⁶

core lengths because the maximum turbulence intensity is found where the potential core closes. Figure 13 shows radial profiles of $u'v'$ correlations. The maximum correlation levels are overpredicted near the nozzle exit, which again is consistent with the higher degree of mixing found in the LES data.

The reason for the underprediction of the length of the potential core region can probably be found in a combination of several factors. The most important are probably subgrid-scale viscosity, numerical scheme, inflow conditions, and entrainment boundary conditions. The large $u'v'$ in the predicted initial jet region indicates that a transition takes place in the shear layer near the nozzle lip (see



a) Jet 1



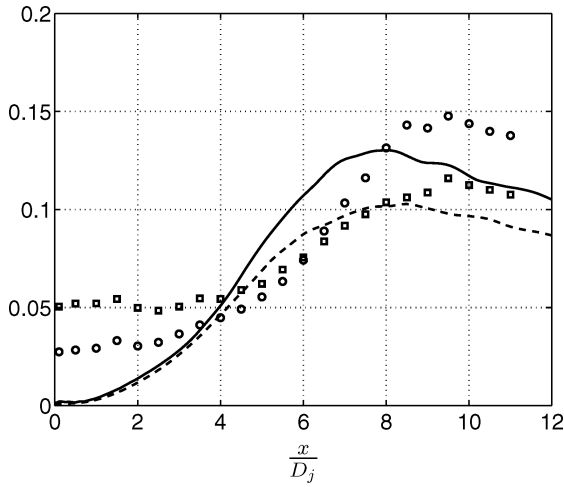
b) Jet 2

Fig. 11 Radial profiles of axial velocity. The profiles have been staggered according to their axial location. See also legend to Fig. 10.

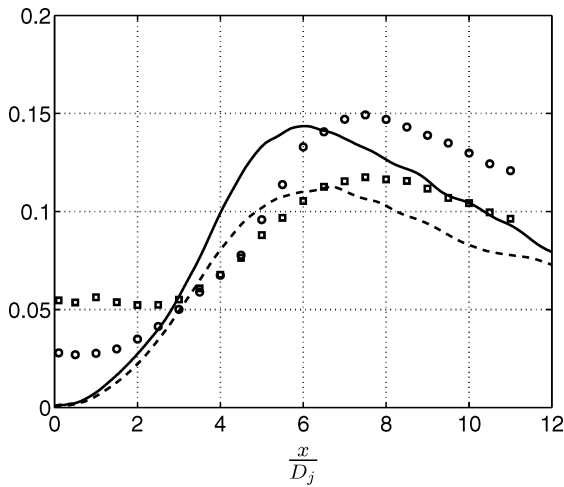
Fig. 13). The fact that the predicted $u'v'$ correlations for locations farther downstream are closer to the measured correlations and that the maximum turbulence intensities are captured by the LES (see Fig. 12), shows that once the transition process is completed the flow is physically correct. However, the overpredicted mixing caused by the initial transition seems to result in a shorter potential region. An increase in the subgrid-scale dissipation would probably decrease the turbulence intensity and delay the transition process and thus change the length of the potential core. Depending on the amount of dissipation, the potential core length could either be shortened or elongated. The effect of an increase of artificial dissipation introduced by the numerical scheme is likely to be similar to that of increased subgrid-scale dissipation.

Bogey and Bailly¹⁰ investigated the effects of the inflow conditions on the jet flow. It was found that the use of a smaller shear-layer momentum thickness resulted in higher turbulence intensities in the shear layer and hence a more rapid transition. In the present study a thinner boundary layer on the nozzle walls would probably imply less overpredicted initial mixing that might affect the length of the potential core. Adding unsteady forcing of incoming disturbances at the domain inlet could possibly affect the transition process and weaken the initial mixing.

Concerning the effects of the entrainment flow, only the correct entrained mass flow will give the correct flow behavior in the hydrodynamic region. Lack of entrained mass will be compensated by backflow, which results in a recirculation zone surrounding the jet and prevents it from spreading correctly. However, the RANS



a) Jet 1



b) Jet 2

Fig. 12 Axial profiles of turbulence intensities: — and ---, u'_{rms} and v'_{rms} , respectively; and $\circ$ and $\square$, corresponding measured¹⁶ quantities.

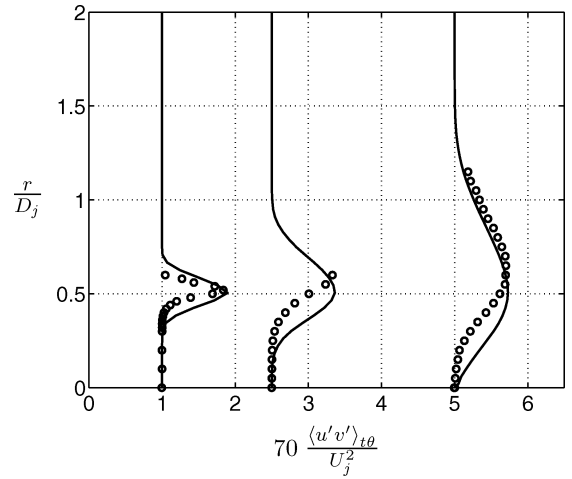
computations from which the boundary entrainment velocities have been interpolated give a somewhat overpredicted potential core length. In summary, the correct potential core length can probably be obtained by modification of the subgrid-scale properties alone. However, a more physical approach would be to modify the inflow conditions to produce results in better agreement with measurements.

B. Two-Point Space-Time Correlations

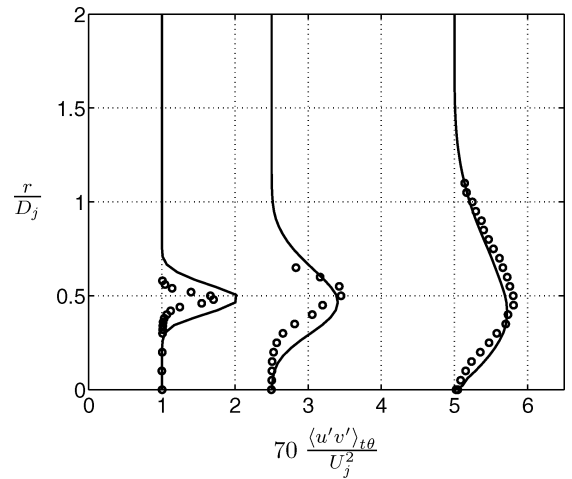
Two-point space-time correlations were obtained for the axial-velocity component in a few shear-layer locations along the nozzle lip line. The two-point space-time correlation of axial velocity for a certain spatial separation ξ and separation in time τ is given by

$$\mathcal{R}_{uu}(\mathbf{x}, \xi, \tau) = \frac{\langle u'(\mathbf{x}, t)u'(\mathbf{x} + \xi, t + \tau) \rangle_t}{\sqrt{\langle u'^2(\mathbf{x}) \rangle_t} \sqrt{\langle u'^2(\mathbf{x} + \xi) \rangle_t}} \quad (19)$$

where u' denotes resolved fluctuation of axial velocity and $\mathbf{x}$ is the position in the flowfield where the two-point correlation is evaluated. Figure 14 shows correlations obtained in the shear layer at an axial location corresponding to the potential core closure. The curve centered at $\tau = 0$ corresponds to the autocorrelation and each consecutive curve is obtained by increasing the spatial separation ξ by $0.3D_j$. For homogeneous isotropic turbulence the correlation peak would be unity for all spatial separations, that is, the increasing deviation from unity with increasing spatial separation is a measure of how far from frozen the turbulence is. The shape of the correlation curves as well as the correlation envelope, that is, the curve connecting the correlation peaks, is in rather good agreement with the



a) Jet 1



b) Jet 2

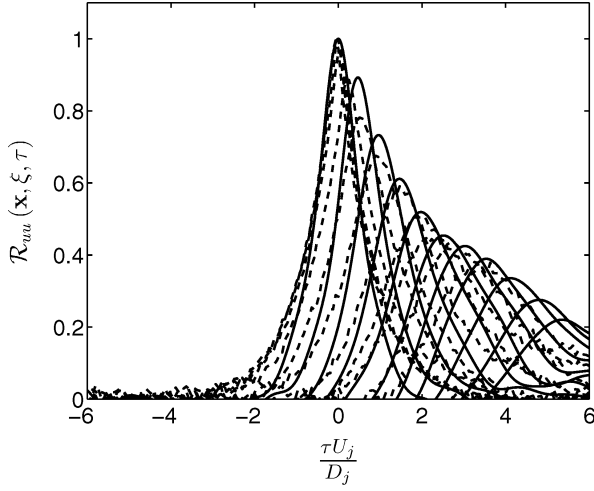
Fig. 13 Radial profiles of predicted and measured $u'v'$ correlation (see also legend to Fig. 10).

measured data. Figure 15 depicts contours of two-point space-time correlations in the $\xi\tau$ plane. Again, the correlations have been obtained in the shear layer at the end of the potential core. The peaks of the predicted correlations are indicated by stars, and those of the correlations obtained from the two-point measurements (Refs. 18 and 19; P. Jordan, private communication, 2003) are indicated by circles. The slope of a line connecting the correlation peaks corresponds to the local convection velocity of turbulence structures.

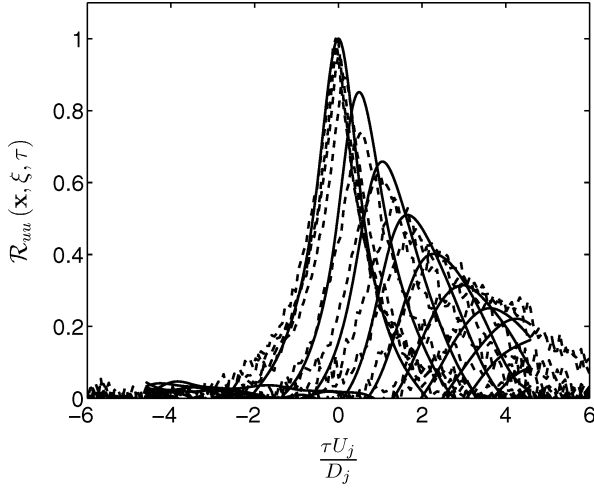
Figure 16 gives a comparison of predicted correlations for jets 1 and 2. The correlations are presented in the shear layer at axial positions normalized with the length of the potential core. As can be seen, there are no great differences in the correlations for the two jets. However, because the potential core is shorter for jet 2, turbulence scales grow faster for this jet.

C. Length and Timescales of Turbulence Structures

Estimates of local integral length and timescales of turbulence structures were obtained by integration of the corresponding autocorrelation and spatial correlation, respectively. The first crossing of the coordinate axis was used as the upper limit for the integration. Figure 17 gives a comparison of length and timescales for jets 1 and 2. In Fig. 17a, the axial development of predicted integral length scale is depicted. To be able to make a comparison of the two jets, the axial coordinate has been normalized by the length of the potential core. Figure 17b shows the axial development of integral timescale. Scales obtained from the two-point measurements (Refs. 18 and 19; Jordan, private communication) are indicated by circles and stars for jets 1 and 2, respectively.



a) Jet 1



b) Jet 2

Fig. 14 Two-point space-time correlations obtained in the shear layer at the end of the potential core ($x = L_c$, $r = 0.5D_j$): —, LES data and ---, measurements (Refs. 18 and 19; Jordan, private communication).

D. Far-Field Sound Pressure Levels

Power spectra of pressure fluctuations and sound pressure levels have been obtained for a few far-field observer locations. The observer locations coincide with the microphones in the experimental setup¹⁷ (Fig. 18). The sound pressure level is here defined as

$$\text{SPL} = 20 \log_{10} \left[\sqrt{\langle (p')^2 \rangle_t} / p_{\text{ref}} \right] \quad (20)$$

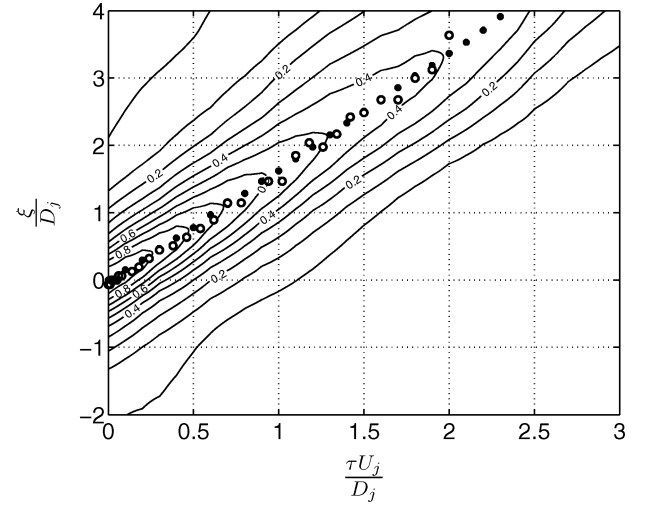
where

$$p_{\text{ref}} = \sqrt{\rho_{\infty} c_{\infty}} 10^{-12} = 2.0 \times 10^{-5} \text{ Pa} \quad (21)$$

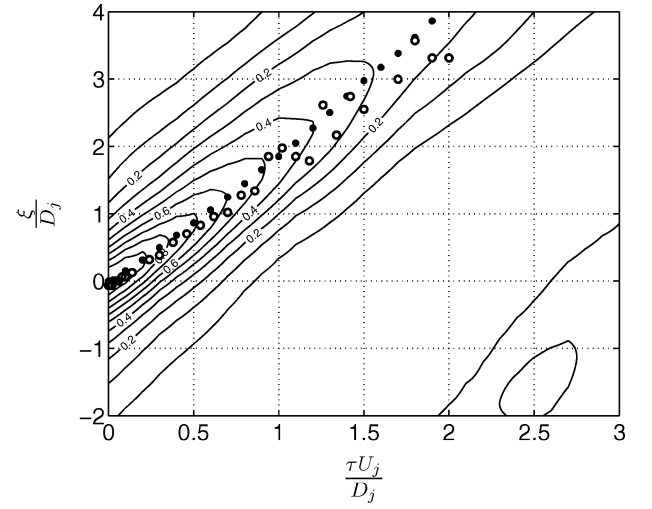
Figure 19 shows a comparison of predicted and measured¹⁷ power spectra for a few observer locations on the inner arc ($r = 30D_j$). To make the comparison with measured data easier, the spectra have been filtered in third octave bands. The spectra have been staggered by multiplying the intensity by a factor $C = 10^{2n}$, where $n = [(\theta - 20)/40]$, θ being the angle from the jet axis (Fig. 18). This amplification corresponds to a $20n$ -dB shift of the spectra. The sound pressure level as function of frequency is obtained as

$$\text{SPL}(f) = 20 \log_{10} \left[\sqrt{C \hat{p}' \hat{p}'^*} / p_{\text{ref}} \right] \quad (22)$$

where $\hat{p}'$ denotes the Fourier transform of the pressure fluctuation and $\hat{p}'^*$ its conjugate. The oscillating nature of the spectra in the low-frequency range is because the length of the time series used is insufficient to fully resolve these low frequencies. For low and high angles, the predicted levels are in good agreement with the experimental data up to at least Strouhal number = 1.5 (see Fig. 19). For



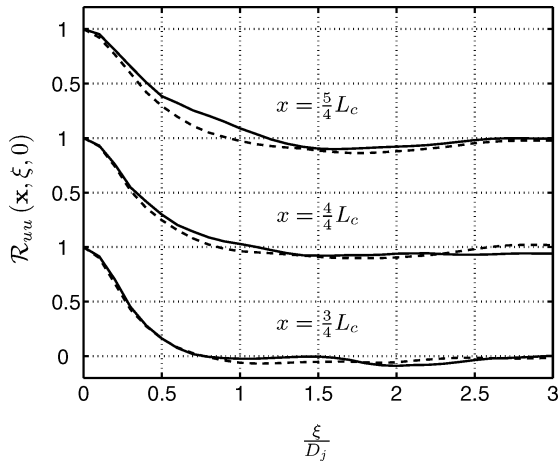
a) Jet 1



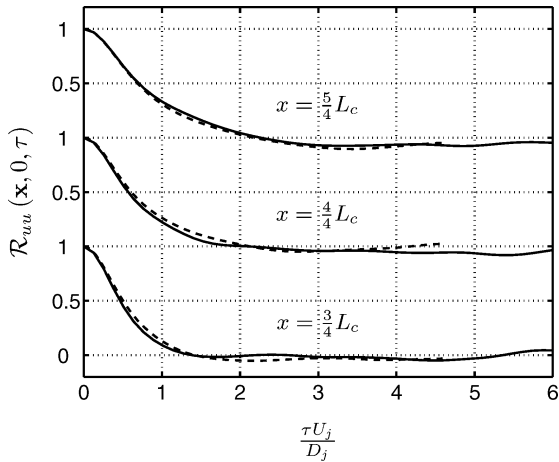
b) Jet 2

Fig. 15 Contours of two-point space-time correlations obtained in the shear layer at the end of the potential core ($x = L_c$, $r = 0.5D_j$). The peaks of the predicted correlations are indicated by stars, and the peaks of the measured data (Refs. 18 and 19; Jordan, private communication) are indicated by circles.

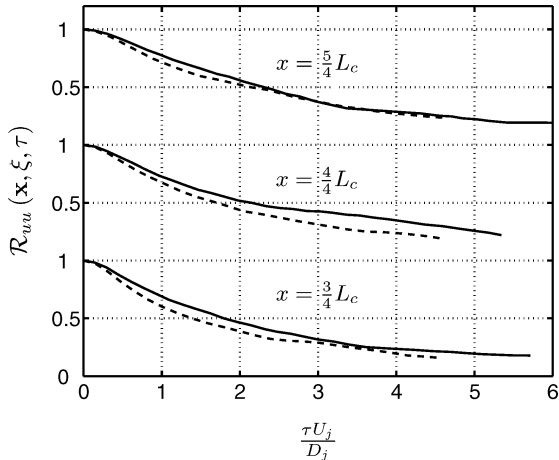
the intermediate angles on the other hand, the predicted amplitude decreases rapidly above $Sr = 1.0$. This is because the grid is not fine enough to support high-frequency acoustic waves in the radial direction. Assuming that it is sufficient to have four cells per wavelength to be able to capture a propagating wave of a certain frequency, the cells in the area where the Kirchhoff surface is located would support propagating waves of Strouhal numbers up to $Sr \approx 1.2$. This means that the mesh itself acts as a low-pass filter as the acoustic waves propagate through the domain. This effect is especially noticeable for the intermediate angles, where the mesh, caused by the resolution demands at the jet exit, is very coarse (see Fig. 5a). The filtering of acoustic waves caused by the coarseness of the mesh might be an explanation for the somewhat overpredicted spectrum amplitudes in the high-frequency range for the observers on the forward arc ($\theta > 90$ deg) (see Fig. 19). This overprediction might be caused by unphysical, high-frequency acoustic waves generated in the hydrodynamic jet region. An indication of such pollution of high frequencies by the presence of spurious noise was found when Lighthill's⁹ acoustic analogy was used for sound propagation, which has been reported in Andersson.²⁹ These high-frequency waves will, for most observers, be damped out by numerical dissipation. However, in the upstream direction the resolution is significantly finer relative to the downstream direction, which means that some of these



a) Spatial correlations



b) Autocorrelations



c) Correlation envelopes

Fig. 16 Comparison of predicted spatial correlations, autocorrelations, and space-time correlation envelope for —, jet 1 and ---, jet 2.

nonphysical waves might reach the Kirchhoff surface and thus contribute to the predicted pressure signal. Figure 20 gives a comparison of predicted and measured SPL for the inner arc. The predicted SPL have been obtained by integration of the corresponding spectra excluding the contribution from frequencies lower than Strouhal number $St = 0.05$. The underprediction of the spectrum amplitude above $St = 1.0$ gives underpredicted SPL for the observers between 50 and 120 deg for jet 1. This effect does not show up in the sound pressure levels for jet 2 though. For jet 2 SPL for the 20-deg observer is underpredicted, which also can be seen in the power spectrum for

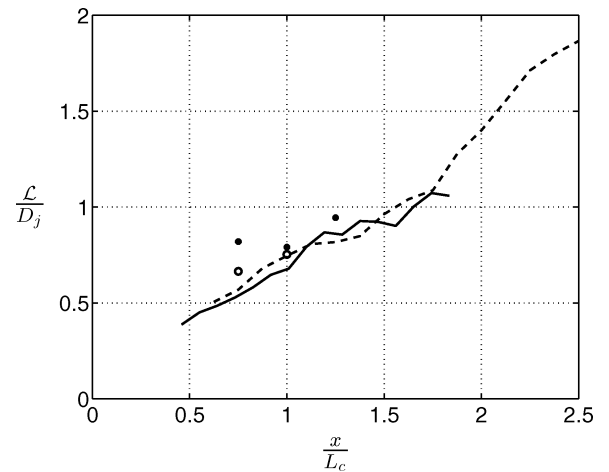
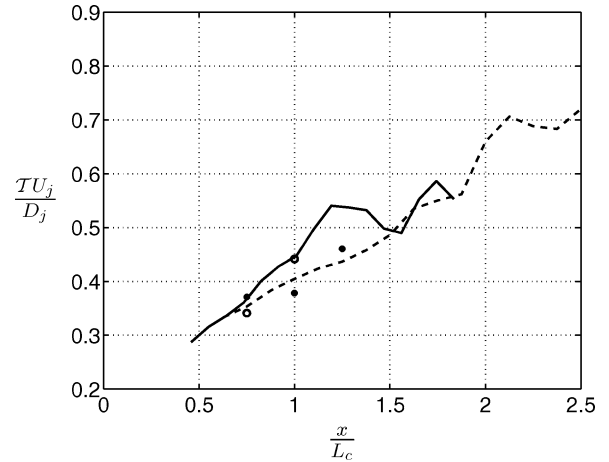
a) Integral length scale $\mathcal{L}$ b) Integral timescale $\mathcal{T}$

Fig. 17 Axial development of integral length and timescale: —, jet 1; ---, jet 2; ○ and □, corresponding measured (Refs. 18 and 19; Jordan, private communication) quantities.

this observer location where the peak level is underpredicted. As can be seen in Fig. 18, the 20-deg observer is very close to the integration surface, which can affect the results. Figure 20 shows that the overall directivity is captured and that a peak in the radiated sound can be found for observers in the region of 20–30 deg from the axis.

Figures 21 and 22 show power spectra and sound pressure levels for the observers on the $50D_j$ arc, respectively. Figure 23 (Refs. 12, 17, and 31–33) gives a comparison of predicted SPL for jet 1 with various sets of measured data obtained for unheated jets. The measured acoustic intensities $I = p'p'$ have been scaled to $M_j = 0.75$ and $r/D_j = 30$ according to

$$I \sim U_j^8 D_j^2 / c_\infty^5 r^2 \quad (23)$$

The predicted SPL shows a clear increase for observation angles greater than 110 deg unlike all of the experimental results represented in Fig. 23. As mentioned earlier, this behavior might be an effect of contamination of the near-field data in the upstream arc. Note that the measurements by Jordan et al.¹⁷ deviate significantly from classical measurements by Mollo-Christensen et al.,³¹ Lush,³² and Tanna¹² near 90 deg; the predicted levels are, however, very close to the results by Jordan et al.¹⁷

There are not very large differences in the sound pressure levels obtained for jet 1 and those obtained for jet 2 (Fig. 24). According to Jiang et al.,³⁴ increasing the jet to ambient temperature ratio results in a weaker sound source but increased sound pressure levels. Tests done to investigate the heating effects on the radiated sound have shown that for high exhaust speeds increasing the jet to ambient

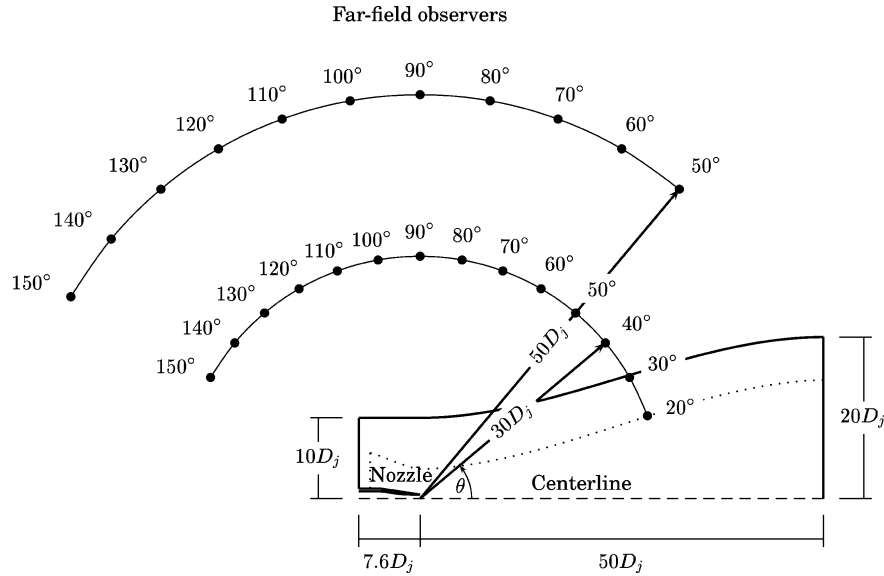
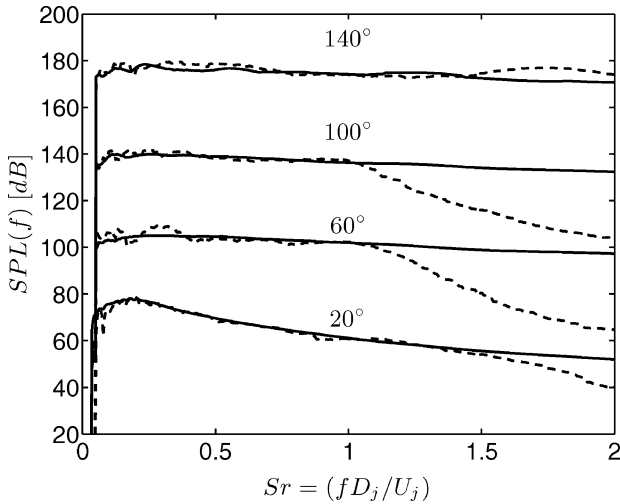
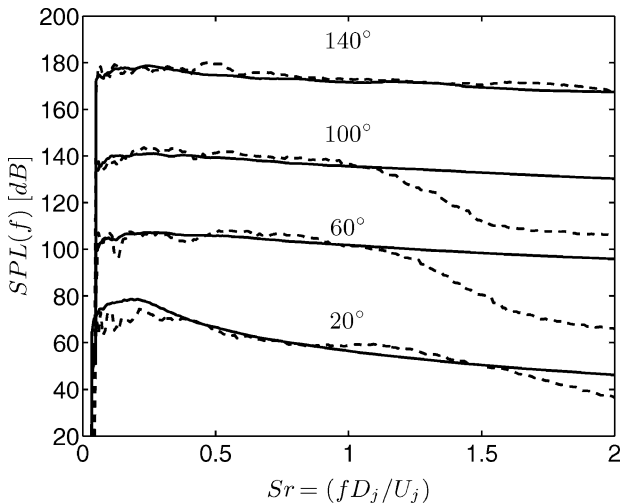


Fig. 18 Acoustic pressure fluctuations were evaluated in 25 observer locations situated on two arcs in the far-field regions of the jet: $\cdots$, Kirchhoff surface.

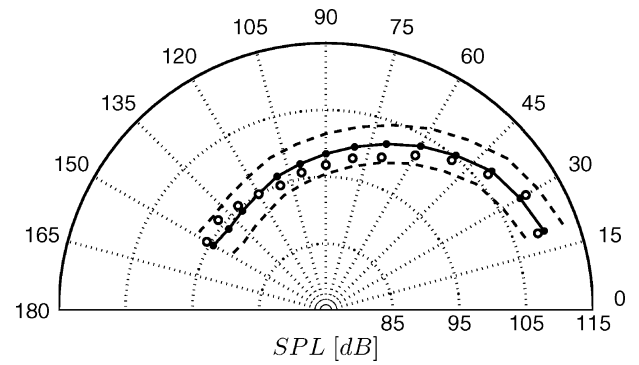


a) Jet 1

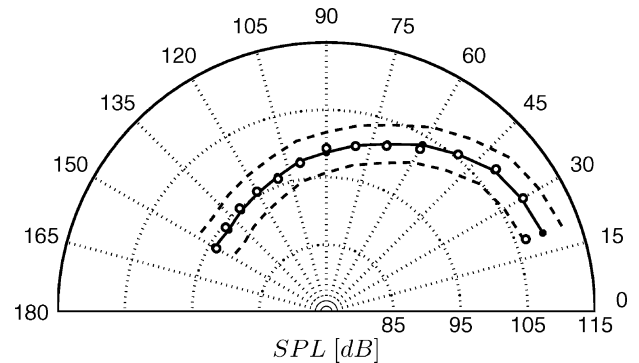


b) Jet 2

Fig. 19 Comparison of $\cdots$, predicted and — , measured¹⁷ power spectra of far-field pressure signal for a few observer locations on the inner arc ($r = 30D_j$). The spectra have been staggered by adding a factor $20n$ [dB] where $n = [(\theta - 20)/40]$, θ being the angle from the jet axis.



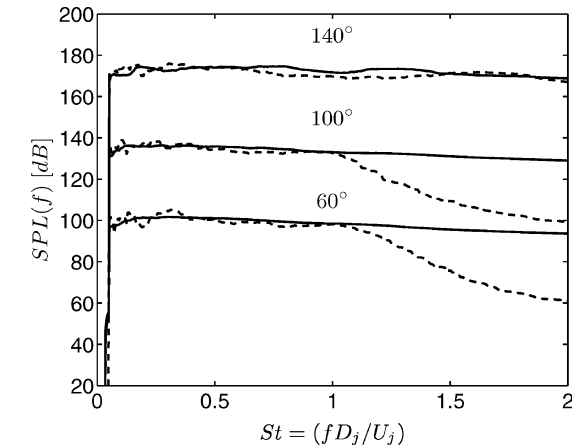
a) Jet 1



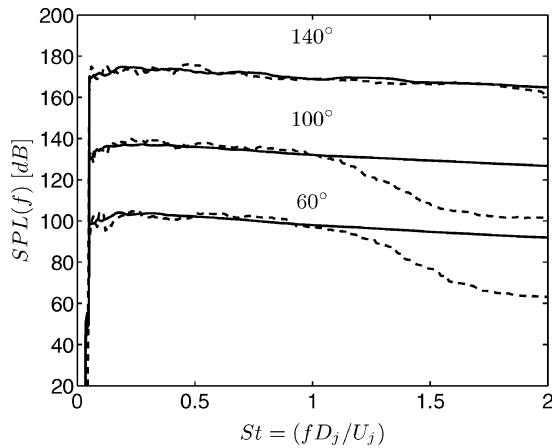
b) Jet 2

Fig. 20 Sound pressure levels in observer locations situated $30D_j$ from the nozzle outlet at different angles θ measured from the jet axis (see Fig. 18): — , measured¹⁷ levels and $\cdots$, measured levels ± 3.0 dB. Predicted SPL obtained using Kirchhoff surface integration are represented by $\circ$.

temperature ratio reduces the noise levels, whereas at lower exhaust speeds the heated jet is the noisier.³⁵ The slight increase in SPL found for jet 2 is, however, not sufficiently large to be able to draw any conclusions regarding the heating effects on radiated sound in this study. Moreover, the measurements by Tanna,¹² where a wide range of exhaust velocities and temperature ratios were tested, indicates that Mach number 0.75 is in the range where the trend changes from increase to decrease of emitted sound as increasing the temperature ratio, and thus the heating effects on the sound pressure levels in the far field should be small at this Mach number. However, although the

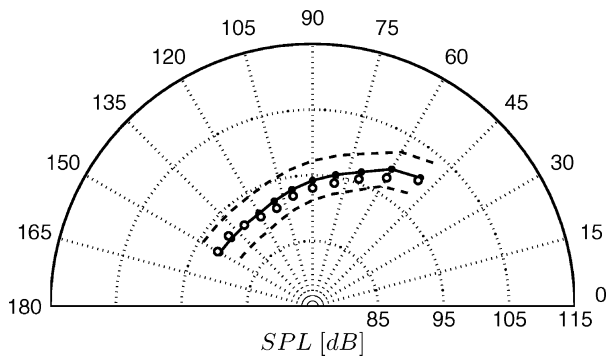


a) Jet 1

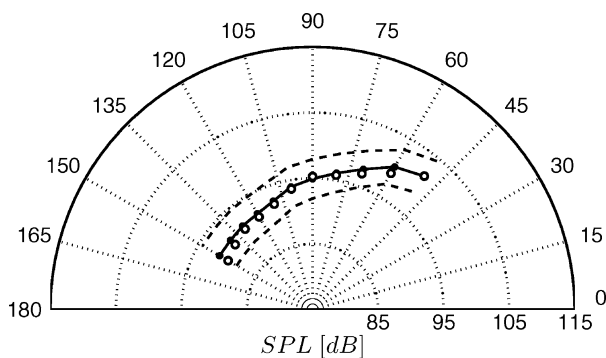


b) Jet 2

Fig. 21 Power spectra of far-field pressure signal for a few observer locations on the outer arc ($r = 50D_j$). See also legend to Fig. 19.



a) Jet 1



b) Jet 2

Fig. 22 Sound pressure levels in observer locations situated $50D_j$ from the nozzle outlet; see also legend to Fig. 20.

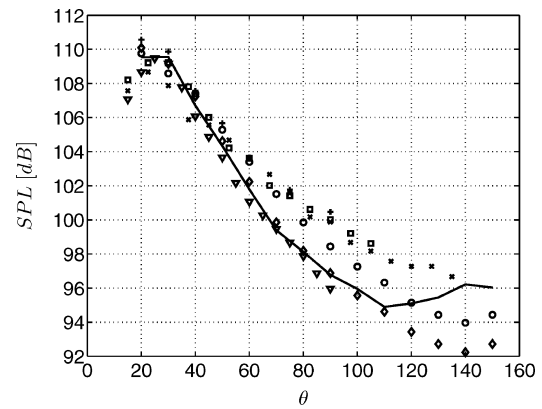


Fig. 23 Sound pressure levels $30D_j$ from the nozzle exit: —, predicted SPL for jet 1. Experiments are denoted as follows: +, Mollo-Christensen et al.³¹ ($M_j = 0.9$, $Re_D = 5.4 \times 10^5$); □, Lush³² ($M_j = 0.88$, $Re_D = 5.0 \times 10^5$); ×, Tanna¹² ($M_j = 0.9$, $Re_D = 9.1 \times 10^5$); ▽, Stromberg et al.³³ ($M_j = 0.9$, $Re_D = 3600$); ○, Jordan et al.¹⁷ ($M_j = 0.75$, $Re_D = 7.5 \times 10^5$); and ◇, Jordan et al.¹⁷ ($M_j = 0.9$, $Re_D = 9.0 \times 10^5$).

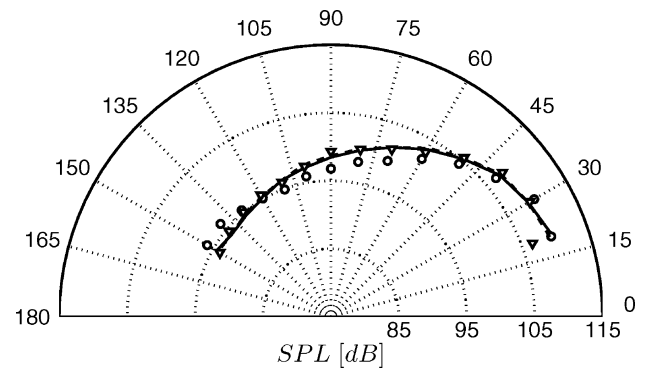
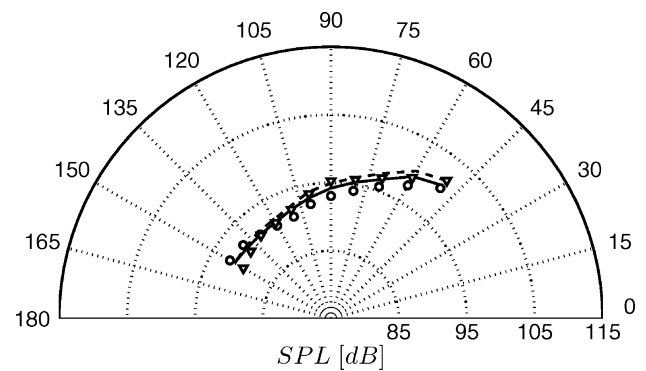
a) $r = 30D_j$ b) $r = 50D_j$

Fig. 24 Comparison of SPL for jets 1 and 2: — and ---, measured¹⁷ data for jets 1 and 2, respectively; ○ and △, predicted SPL.

changes in overall levels are virtually not affected by the temperature ratio for the current Mach number, the work by Tanna¹² indicates that there are significant changes in the spectra: the amplitude at very low frequencies increases when the temperature ratio is increased while the amplitude in the high-frequency range decreases. For the current Mach number these two effects cancel, and therefore no changes are visible in the overall levels. Unfortunately, these spectrum effects are not captured because the high-frequency range is filtered by the numerics and the low-frequency range is unresolved.

In summary, the predicted sound pressure levels are for all observers within the 3.0-dB deviation from the measured levels indicated by dashed lines in Figs. 20 and 22 and for most of the observers within a 1.0-dB deviation, which must be considered as a satisfying result. These levels have been obtained directly from the LES/Kirchhoff data without adjustment. Also, the predictions are

obtained for a lower Reynolds number than the measured data, which probably affects the acoustic signature. However, the Reynolds-number effects would probably be most significant in the high-frequency range, which is highly filtered by the numerical method anyway.

VII. Conclusions

Large-eddy simulation of a compressible Mach 0.75 nozzle/jet configuration has been performed. The Reynolds number based on jet velocity and nozzle diameter was 5.0×10^4 . Jet simulations were performed at two heating conditions, an unheated jet where the exhaust static temperature was equal to the static temperature of the surrounding air and a heated jet where the nozzle outlet temperature was twice that of the ambient air. Profiles of time and azimuthally averaged flow properties have been compared with data measured by Jordan et al.¹⁶ Although some deviations occur, the results are in general in very good agreement with experiments. The good results are probably attributed to the homogeneity of the mesh used and the fact that a nozzle geometry was included in the calculation domain. Using a combination of Cartesian and polar mesh blocks makes it possible to ensure that cells in the outer regions of the calculation domain are not too large and prevents cells from being clustered in the centerline region.

The maximum levels of turbulence intensities are well captured for both jets. Furthermore, the turbulence anisotropy of the jets is correctly predicted. The initial jet spreading and the potential core lengths are, however, not predicted correctly. Overpredicted $u'v'$ correlations close to the nozzle exit indicate that the degree of mixing is too high, which makes the transition process too efficient, and hence the length of the potential region becomes shorter. This might be caused by a combination of several factors such as lack of subgrid-scale dissipation, the numerical method used, entrainment boundary conditions, and inflow conditions. Of these, the inflow conditions are believed to be the most important.

Two-point space-time correlations were obtained in a number of locations in the shear layer along the nozzle lip line. Correlation curves obtained at $x = L_c$ were in good agreement with experiments.¹⁸ Two-point space-time correlations in the shear layer were used to obtain estimates of integral length scales, integral timescales, and eddy convection velocities. The agreement with corresponding quantities obtained from the measured data was satisfying. Only small differences could be identified in correlations and integral scales for the two jets.

Sound pressure levels in the far field were evaluated with a hybrid approach. Kirchhoff surface integration was utilized for the propagation of sound to far-field locations. Predicted sound pressure levels were in excellent agreement with the levels measured by Jordan et al.¹⁷ for both jets. However, the mesh used fails to capture waves of Strouhal numbers higher than $Sr \approx 1.2$. This results in a rapid fall-off of the spectrum amplitudes for higher Strouhal numbers. The lower part of the spectra is accurately captured, however. Because the sound pressure levels obtained were in good agreement with the measured levels, capturing the lower part of the spectra seems to be sufficient to represent the main part of the radiated sound. Both measurements and predictions show slightly higher SPL for the heated jet. The increase in SPL is, however, too small to be able to draw any conclusions regarding the effects of the increased temperature ratio on the radiated sound.

Although, the predictions of both near-field statistics and far-field acoustic signature are generally in good agreement with measured data, there are a number of parameters to which the predictions are sensitive. The fact that the predictions have been obtained for a lower Reynolds number than the measured data changes the range of scales present in the jet. This will affect the frequency content of the radiated sound. The inflow conditions affect the initial mixing, and thus the initial shear-layer growth and the potential core lengths are affected. Moreover, because the current effort deals with a subsonic jet, most of the radiated sound is generated in the potential core region, which is highly dependent of the nature of the incoming disturbances. However, both the Reynolds-number effects and the effects of the inflow conditions on the radiated sound are probably

noticeable mostly in the high-frequency range, which is significantly filtered by the numerical method. Another uncertainty is related to the location of the Kirchhoff surface. The predicted sound pressure levels are sensitive to the location of the integration surface. The surface has to be placed far enough from the jet to avoid hydrodynamic fluctuations in the surface region but close enough to avoid numerical dissipation of the radiated sound. To accomplish a good compromise, a jet-fitted integration surface has been used. Still the surface location should be considered as a factor of uncertainty.

With a surface integral approach, it is sufficient to use a mesh resolving the main noise generation structures and supporting acoustic waves within the frequency range contributing most to the overall sound pressure levels. However, if the aim were to capture the sound spectra in far-field locations, higher mesh resolution and probably higher-order schemes would be desirable.

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Paper III

Effects of Inflow Conditions and Subgrid Model on LES for Turbulent Jets

By

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Effects of Inflow Conditions and Subgrid Model on LES for Turbulent Jets

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The turbulent mixing process prescribing the spreading rate of the jet and the length of the potential core region is influenced by a number of factors. Using large-eddy simulation (LES), the four factors that are believed to be the most important in this respect are: subgrid-scale properties, the accuracy of the numerical scheme, the entrainment boundary conditions, and the inflow conditions. In a previously performed study of a subsonic (Mach 0.75) jet, the turbulence mixing was found to be too efficient and hence the length of the potential core region was underpredicted. In that study indications were found of that the overpredicted mixing was due to the inflow conditions. For a model nozzle, capturing the initial turbulent shear flow might not be of that great importance for accurate prediction of radiated sound since most of these effects will appear in the high-frequency range. When dealing with real engine geometries, however, it becomes quite important. Moreover, methods for industrial use have to cope with complex geometries and high temperature and velocity ratios making the ability to capture the initial flow physics even more important.

In the present work LES has been used for the same Mach 0.75 jet. The acoustic field is extracted to the far field using Kirchhoff surface integration. The effects of inflow conditions, Reynolds number, and subgrid-scale model on flowfield and acoustic signature are investigated.

The Favre-filtered Navier-Stokes equations were solved using a finite-volume method solver with a low-dissipation third-order upwind scheme for the convective fluxes, a second-order centered difference approach for the viscous fluxes and a three-stage second-order Runge-Kutta technique in time. The computational domain was discretized using a block-structured boundary-fitted mesh with approximately 3.0×10^6 cells. The calculations were performed on a parallel computer, using message-passing interface (MPI). A compressible form of Smagorinsky's subgrid-scale model was used to compute the subgrid-scale stresses. Absorbing boundary conditions based on characteristic variables were adopted for all free boundaries.

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Nomenclature

Latin symbols

c	speed of sound
$\mathcal{C}_{ij}$	Cross stress
C_p	specific heat at constant pressure
C_R, C_I	Smagorinsky model coefficients
e	energy
k	kinetic energy
$\mathcal{L}_{ij}$	Leonard stress
p	pressure
Pr	Prandtl number
Q	state vector
q_j	energy diffusion vector
r	radial coordinate or distance from source to observer
$\mathcal{R}_{ij}$	Reynolds stress
Re_D	Reynolds number based on the jet diameter
S	cell face area
S_{ij}	strain rate tensor
T	temperature
t	time
u_i	Cartesian components of velocity vector
$\mathbf{x}$	flowfield location
x_i	Cartesian coordinate vector component
$\mathbf{y}$	far-field location

Greek symbols

Δ	filter width
δ_{ij}	Kronecker delta
γ	gas constant
μ	dynamic viscosity
ρ	density
σ_{ij}	viscous stress tensor
τ_{ij}	subgrid-scale stress tensor
τ_r	retarded time
ε	turbulence dissipation rate

Subscripts

0	total condition
∞	freestream or ambient conditions
j	jet, nozzle-exit condition
rms	root-mean-square
t	turbulent quantity

Superscripts

"	unresolved fluctuation
—	spatially filtered quantity
'	resolved fluctuation
$\sim$	spatially Favre-filtered quantity
$\wedge$	Fourier transform

SGS subgrid scale

Symbols

$\langle \dots \rangle_\theta$ circumferentially averaged quantity
 $\langle \dots \rangle_t$ time-averaged quantity

I. Introduction

THE number of commercial aircraft in service is continuously growing and airports around the world are growing in size, which increases exposure to air traffic noise in populated areas. Threshold values for noise certification of new aircraft are based on global restrictions on noise generated by air traffic. Moreover, local restrictions for air ports with heavy traffic limit operating hours or even impose direct noise penalty costs. Stricter regulations on noise levels in the surroundings of airports have made the abatement of near-ground operation noise an important issue for aircraft and engine manufacturers, and noise generation has now become an important design factor that is taken into consideration early in the construction process.

Using a grid fine enough in the far-field regions to minimize the introduction of sound propagation errors, the acoustic field can be obtained directly from the flowfield simulation. This requires a detailed numerical compressible flow simulation, e.g. direct numerical simulation (DNS), see for example Freund¹ and Mitchell *et al.*,² or large-eddy simulation (LES), as in e.g. Bogey *et al.*^{3,4,5,6} and Mankbadi *et al.*⁷ In DNS, all scales of the turbulent flowfield are computed accurately, which requires a mesh fine enough to capture even the smallest scales of the flow, whereas in LES only the large scales of the flow are resolved and the influence on these large scales of the smaller, unresolved scales is modeled using a subgrid-scale model. With the computational resources available today, DNS is restricted to fairly simple geometries and low Reynolds number flows. Moreover, it is believed, see Mankbadi,⁸ that large scales are more efficient than small ones in generating sound, which justifies the use of LES for sound predictions. To save computational time, a hybrid approach may be used in which the computational problem is divided into two parts. An LES can be used to obtain the unsteady non-linear near field, which in the jet noise case corresponds to the hydrodynamic jet region. The acoustic field is then extended to far-field observer locations using, for example, Kirchhoff⁹ surface integration, see Lyrantzis¹⁰ and Freund *et al.*¹¹ Using a hybrid approach may even lead to noise predictions of higher quality than a direct approach since the amount of numerical dissipation to which the sound waves are exposed is decreased. Furthermore, the frequency cut-off is often higher in the integration surface regions than in the far field.^{12,13}

Andersson *et al.*^{14,15,16} used LES to study the flow and radiated sound of a subsonic (Mach 0.75) jet at different heating conditions. In these studies the initial turbulence mixing was found to be too efficient, which resulted in underprediction of the potential core lengths. The predicted acoustic signature was in generally good agreement with experimental data though. For the forward arc observers the sound pressure levels (SPL) were, however, somewhat overpredicted, which is believed to be due to high-frequency noise generated in the initial jet region as a consequence of the overpredicted turbulent mixing.

The reason for the underprediction of the length of the potential core region can probably be found in a combination of several factors. At the time when the present study was initiated the factors believed to be the most important were: subgrid-scale viscosity; numerical scheme and resolution; entrainment boundary conditions and inflow conditions. DeBonis *et al.*¹⁷ reported an underprediction of the potential core region when using LES for the simulation of a supersonic jet. It was discussed in that work that the reason for the underprediction was a lack of subgrid dissipation. Furthermore, it has been mentioned by Morris *et al.*¹⁸ that changes in the subgrid-scale model constants affect the potential core length and the initial growth of the jet. DeBonis¹⁹ found that the potential core region could be extended by increasing the subgrid-scale dissipation. This might seem to be a quite natu-

ral effect since the increased subgrid-scale viscosity decreases the intensity of the resolved turbulent fluctuations and hence the shear-layer mixing becomes less efficient and that this in turn leads to an elongated region of potential flow. However, as will be shown in the present study, this is not always the case. Although the correct potential core length might be obtained by modification of the subgrid-scale properties alone, a more physical approach would be to modify the inflow conditions to produce results in better agreement with measurements. The shape of the velocity profile and turbulence level at the nozzle exit might be factors that are important for the development of the jet. Thin boundary layers at the nozzle exit results in steep velocity gradients in the initial part of the shear layer and thus a more violent encounter with the irrotational surrounding fluid. This may result in larger fluctuations and thus also higher degree of turbulent mixing. Bogey *et al.*⁵ investigated the effects of the inflow conditions on the jet flow. It was found that the use of a thinner shear-layer momentum thickness resulted in higher turbulence intensities in the shear-layer and hence a more rapid transition. In the present study the effect of specifying a velocity profile at the nozzle inlet and that of adding synthesized turbulence at the inlet boundary has been investigated.

Tucker²⁰ used a scheme for the convective fluxes based on a weighted average of centered and upwind biased components. It was found that even with a weak blending of fifth-order upwinding with a fourth order central scheme, i.e. only small contributions from the upwind biased components, the effects of the upwind scheme is still evident. However, the effect of the artificial dissipation introduced by the numerical scheme ought to be the same as that of the subgrid-scale dissipation, i.e. more numerical dissipation should lead a delayed shear layer transition. Consequently, the numerical scheme used in Refs. 14–16 does probably not introduce too much artificial dissipation since the turbulent mixing is overpredicted. Concerning the effects of the entrainment flow, only the correct mass flow will give the correct flow behavior in the hydrodynamic region. A lack of entrained mass will be compensated by a back flow, which results in a recirculation zone surrounding the jet preventing it from spreading correctly.

The present work has been conducted within the EU 5th framework program JEAN^a which was an European project investigating the physics behind noise generation in single-stream jets. In the continuation of this project, CoJeN^b, the flowfield and radiated sound of a high-subsonic coaxial nozzle/jet configuration is being investigated. The objective of CoJeN is to develop and validate prediction tools that can be used by the aerospace industry to assess and optimize jet noise prediction techniques. The aim is to provide design tools that can be used to develop low-noise nozzles for high bypass ratio (HBR) engines. For prediction techniques to be useful to industry, the methods must cope with realistic jet flows such as coaxial jet configurations with high velocities, significant velocity and temperature gradients, and arbitrary nozzle geometries. Hence, the ability to capture the initial flow physics becomes even more important.

II. Overview of the Present Study

The effects of changes in inflow conditions, Reynolds number, and subgrid-scale model on the flow in the initial jet region and on the radiated sound of an isothermal Mach 0.75 jet have been investigated. The results are validated against near-field and far-field measurements obtained by Jordan *et al.*^{21,22,23} These measurements have also been reported in part by Power *et al.*²⁴ A simulation of a corresponding jet previously reported by Andersson *et al.*^{14,15,16} is used as a reference. This reference simulation is referred to as *Jet I*. In the present study the results of six simulations, *Jet II* – *Jet VII*, of the single-stream jet will be reported. Table 1 gives more detail on these simulations in terms of modifications made from the *Jet I* simulation. In the reference simulation the Reynolds number was decreased from approximately 9.0×10^5 , which was the Reynolds number in the measurements, to 5.0×10^4 by

^aJEAN – Jet Exhaust Aerodynamics & Noise, Contract number: G4RD-CT-2000-000313

^bCoJeN – Computation of Coaxial Jet Noise, contract number: AST3-CT-2003-502790.

modifying the viscosity on the assumption that the flow is only weakly Reynolds number dependent. To justify this assumption, a simulation was made (*Jet II*) with the Reynolds number for which the measurements were obtained. For this simulation wall functions were used near the nozzle walls. Obviously, this is a rather coarse LES since no changes were made to the mesh and this is a fact that should be considered when analyzing the results of this simulation. The Reynolds number in *Jet III* to *Jet VII* is the same as in the reference simulation. In *Jet III*, the inlet conditions have been modified by adding synthesized turbulent velocity fluctuations. In *Jet IV*, the numerical accuracy has been improved by adding a flux correction term that approximates the effect of the Leonard stresses that appears in the spatially filtered Navier-Stokes equations. This flux correction technique will be described in more detail in Section IV-C. *Jet V* – *Jet VI* are both simulations where the subgrid-scale dissipation has been modified by changing the filter width. In *Jet VII*, a profile for total pressure has been specified at the nozzle inlet boundary so as to generate thicker boundary layer at the nozzle exit and thus change the shear layer behavior in the initial part of the jet.

Table 1. Jet flow simulations

<i>Jet I</i>	Reference simulation $U_j/c_\infty = 0.75$, $Re_D^* = 5.0 \times 10^4$, $T_j/T_\infty = 1.0$
<i>Jet II</i>	Reynolds number corresponding to the experimental Reynolds number $Re_D = 9.0 \times 10^5$ (wall functions are used near the nozzle walls)
<i>Jet III</i>	Synthesized inlet turbulence based on random Fourier modes ²⁵
<i>Jet IV</i>	Introduction of a Leonard flux correction technique that approximates the effect of the Leonard stresses ($\mathcal{L}_{ij} = \widetilde{\tilde{u}_i \tilde{u}_j} - \tilde{u}_i \tilde{u}_j$)
<i>Jet V</i>	Increased subgrid-scale viscosity by increasing the filter width in the subgrid-scale model ($\Delta = \sqrt{2}(\Delta_1 \Delta_2 \Delta_3)^{1/3}$)
<i>Jet VI</i>	Decreased subgrid-scale viscosity by decreasing the filter width in the subgrid-scale model ($\Delta = \min(\Delta_1, \Delta_2, \Delta_3)$)
<i>Jet VII</i>	Specified inlet profile ($P_0 = P_0(r)$)

* The Reynolds number was decreased using modified viscosity

A. Aerodynamic and Acoustic Measurements Used for Validation

The measurements were made within the JEAN project by Jordan *et al.*^{21,22,23} at the MARTEL facility of CEAT (Centre d'Etudes Aérodynamiques et Thermiques), Poitiers, France. Two-component single-point and mono-component two-point measurements were made using Laser Doppler Velocimetry (LDV). The acoustic field was sampled using an arc of microphones at 30 jet diameters and 50 jet diameters from the jet exit, respectively. For greater detail on the experimental set-up see Jordan *et al.*^{21,22} and Jordan & Gervais.²³

III. Governing equations

The equations solved are the spatially Favre-filtered continuity, momentum and energy equations,

$$\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial (\bar{\rho} \tilde{u}_i)}{\partial x_i} = 0 \quad (1)$$

$$\frac{\partial(\bar{\rho}\tilde{u}_i)}{\partial t} + \frac{\partial(\bar{\rho}\tilde{u}_i\tilde{u}_j)}{\partial x_j} = -\frac{\partial\bar{p}}{\partial x_i} + \frac{\partial\bar{\sigma}_{ij}}{\partial x_j} + \frac{\partial\tau_{ij}}{\partial x_j} \quad (2)$$

$$\begin{aligned} \frac{\partial(\bar{\rho}\tilde{e}_0)}{\partial t} + \frac{\partial(\bar{\rho}\tilde{e}_0\tilde{u}_j)}{\partial x_j} = & -\frac{\partial\bar{p}\tilde{u}_j}{\partial x_j} + \\ & + \frac{\partial}{\partial x_j} \left(C_p \frac{\mu}{Pr} \frac{\partial\tilde{T}}{\partial x_j} + q_j \right) + \frac{\partial}{\partial x_j} (\tilde{u}_i (\bar{\sigma}_{ij} + \tau_{ij})) \end{aligned} \quad (3)$$

where $\bar{\sigma}_{ij}$ and τ_{ij} are the Favre-filtered viscous stress tensor and subgrid-scale viscous stress tensor, respectively. These are here defined as

$$\bar{\sigma}_{ij} = \mu \left(2\tilde{S}_{ij} - \frac{2}{3}\tilde{S}_{mm}\delta_{ij} \right) \quad (4)$$

$$\tau_{ij} = \mu_t \left(2\tilde{S}_{ij} - \frac{2}{3}\tilde{S}_{mm}\delta_{ij} \right) - \frac{2}{3}\bar{\rho}k^{SGS}\delta_{ij} \quad (5)$$

where k^{SGS} is the subgrid-scale kinetic energy

$$k^{SGS} = C_I \Delta^2 \tilde{S}_{mn} \tilde{S}_{mn} \quad (6)$$

μ_t the subgrid-scale dynamic viscosity

$$\mu_t = C_R \bar{\rho} \Delta^2 \sqrt{\tilde{S}_{mn} \tilde{S}_{mn}} \quad (7)$$

and $\tilde{S}_{ij}$ is the Favre-filtered strain rate tensor given by,

$$\tilde{S}_{ij} = \frac{1}{2} \left(\frac{\partial\tilde{u}_i}{\partial x_j} + \frac{\partial\tilde{u}_j}{\partial x_i} \right) \quad (8)$$

The subgrid heat flux appearing in the Favre-filtered energy equation is modeled using a temperature gradient approach

$$q_j = C_p \frac{\mu_t}{Pr_t} \frac{\partial\tilde{T}}{\partial x_j} \quad (9)$$

The filter-width in Eqs. (6) and (7) is the local grid cell width, i.e. $\Delta = (\Delta_1 \Delta_2 \Delta_3)^{1/3}$. It should be noted that this quantity was changed in the *Jet IV* and *Jet V* simulations. The subgrid-scale model used in this work is the Smagorinsky part of the model proposed by Erlebacher *et al.*²⁶ for compressible flows. Constants C_R and C_I that appear in Eqs. (6) and (7) are the Smagorinsky model constants, of which the latter is a constant in the compressibility correction term. The constants are here given by

$$\begin{cases} C_R &= 0.012 \\ C_I &= 0.0066 \end{cases} \quad (10)$$

The system of governing equations, Eqs. (1–3), is closed by making assumptions on the thermodynamics of the gas considered. It is assumed that the gas is thermally perfect, i.e. it obeys the gas law. Furthermore, the gas is assumed to be calorically perfect, which implies that internal energy and enthalpy are linear functions of temperature.

IV. Method

A. Numerical Scheme

The Favre-filtered Navier-Stokes equations were solved using a finite volume method solver with a third-order low-dissipation upwind scheme for the convective fluxes and a centered difference approach for the diffusive fluxes. The temporal derivatives were estimated using a second-order three-stage Runge-Kutta technique. The solver used is based on the G3D family of codes developed by Eriksson.²⁷ The convective scheme is a combination of centered and upwind biased components that have been used with good results for free shear flows by Mårtensson *et al.*²⁸ and, more recently, for shock/shear-layer interaction by Wollblad *et al.*²⁹ The coefficients of low-dissipation upwind scheme for estimation of the convective flux over a cell face are derived using a third-order polynomial $Q(x)$ to represent the variation of the flow state in the direction normal to the face as

$$Q(x) = A + Bx + Cx^2 + Dx^3 \quad (11)$$

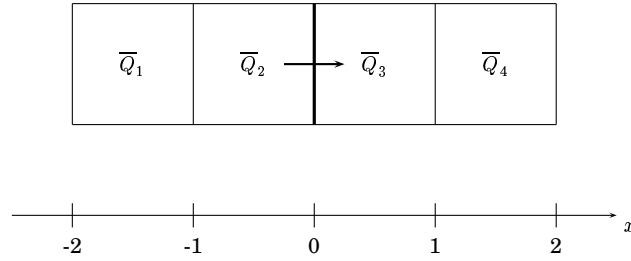


Figure 1. The face state, Q_0 , is estimated using the state in four neighboring cells.

The face state is evaluated using the interpolated value $Q(0)$, modified to include upwinding by adding the third derivative of $Q(x)$ according to

$$\begin{aligned} Q_0 &= Q(0) + \delta Q'''(0) = A + 6\delta D = \\ &= C_1 \bar{Q}_1 + C_2 \bar{Q}_2 + C_3 \bar{Q}_3 + C_4 \bar{Q}_4 \end{aligned} \quad (12)$$

where coefficient δ in front of the upwind term has been chosen by numerical experiments²⁸ to be $(1/96)$ in order to introduce only a small amount of upwinding and thus ensure as high numerical accuracy as possible. The result is the low-dissipation third-order upwind scheme with the coefficients

$$\begin{cases} C_1 = - \left(\frac{1}{12} + \delta \right) = -\frac{9}{96} \\ C_2 = \left(\frac{7}{12} + 3\delta \right) = \frac{59}{96} \\ C_3 = \left(\frac{7}{12} - 3\delta \right) = \frac{53}{96} \\ C_4 = - \left(\frac{1}{12} - \delta \right) = -\frac{7}{96} \end{cases} \quad (13)$$

For more detail on the numerical scheme, see Eriksson²⁷ or Andersson.¹⁶

B. Synthesized Inlet Turbulence

In the *Jet III* simulation, synthesized turbulence was added at the nozzle inlet in the form of fluctuating velocity components. In each solver time step new turbulence velocity components for locations at the nozzle inlet plane are generated using a turbulence synthesize method based on random Fourier modes

and levels of turbulence kinetic energy, k , and turbulence dissipation rate, ε , specified for the nozzle inlet. A short description of the synthesis methodology will be given in this section. Greater detail on the method is given in the PhD thesis by Billson²⁵ or in Billson *et al.*^{30,31,32}

For each location, velocity components v_i are generated as a sum of 30 Fourier modes as

$$\mathbf{v}(\mathbf{x}) = 2 \sum_{n=1}^{30} \hat{u}_n \cos(\mathbf{k}_n \cdot \mathbf{x} + \psi_n) \sigma_n \quad (14)$$

where $\hat{u}_n$, ψ_n and σ_n are, respectively, the amplitude, phase and direction of the n^{th} Fourier mode. The direction is defined by three angles in wave-space: φ_n , α_n and θ_n . These angles and the phase, ψ_n , are chosen randomly for each Fourier mode. The direction σ_n is a vector with length 1.0. The amplitude of each mode $\hat{u}_n$ is computed from a model turbulence energy spectrum function, $E(k_n)$, corresponding to the energy spectrum of isotropic turbulence as

$$\hat{u}_n = \sqrt{E(k_n) \Delta k_n} \quad (15)$$

where Δk_n is a small interval in the spectrum located at k_n . The parameters in the model spectrum are obtained from the values of k and ε specified at the boundary. In this way the sum of $\hat{u}_n^2$ over all n equals the specified turbulence kinetic energy. The inlet boundary values of k and ε were obtained assuming a 5% turbulence intensity and a turbulence length scale corresponding to $0.2D_{nozzle}$.

The velocity fluctuations, u_i , added to the inlet mean velocity are obtained from the synthesized fluctuations, v_i , as

$$u_i^m = a u_i^{m-1} + b(v_i^m + v_i^{m-1}) \quad (16)$$

where m denotes time step and a and b define a low-pass filtering of the synthesized velocity components and are here defined as $a = \exp^{-\Delta t/\tau}$ and $b = \sqrt{(1.0 - a)/2}$, respectively. The time scale, τ , that appears in the denominator of the exponent in the expression for a corresponds to convection of frozen turbulence and is defined as $\tau = l_t/U_c$ where l_t is a turbulence length scale and U_c is a convection velocity representative for the nozzle inlet plane. The use of a low-pass filter makes the effective number of Fourier modes represented greater than the 30 modes used in each turbulence realization.

C. Leonard Flux Correction Implementation

In the *Jet IV* simulation, a flux correction term that approximates the effects of the Leonard stresses has been implemented. The Leonard stresses appears when the convective terms in the momentum and energy equations are filtered spatially. A spatial Favre filter is used in this work, i.e. a mass weighted spatial filter, leading to governing equations on the form given by Eqs. 2–3. Favre filtering is a common filtering approach for compressible flows since it results in governing equations in a convenient form. Moreover, using a filter approach in which fluctuations in density are considered would lead to more complicated subgrid-scale terms. With such a filter, subgrid-scale terms would appear in the continuity equation, terms that would need modeling of their own.³³ Using Favre filtering, the flow properties are decomposed as follows

$$\Phi = \tilde{\Phi} + \Phi'' \quad (17)$$

where $\tilde{\Phi}$ is a Favre-filtered, resolved quantity and Φ'' is the unresolved part of Φ . The Favre-filtered part of Φ is obtained as follows

$$\tilde{\Phi} = \frac{\overline{\rho \Phi}}{\bar{\rho}} \quad (18)$$

in contrast to Favre time averaging

$$\tilde{\Phi} \neq \bar{\Phi} \quad (19)$$

and hence

$$\widetilde{\Phi''} \neq 0 \quad (20)$$

Applying the Favre filtering procedure to the convective term in the momentum equation gives

$$\begin{aligned} \frac{\partial \overline{\rho u_i u_j}}{\partial x_j} &= \frac{\partial}{\partial x_j} \left(\overline{\rho (\tilde{u}_i + u_i'') (u_j + u_j'')} \right) = \frac{\partial}{\partial x_j} \left(\overline{\rho \tilde{u}_i \tilde{u}_j} + \overline{\rho \tilde{u}_i u_j''} + \overline{\rho u_i'' \tilde{u}_j} + \overline{\rho u_i'' u_j''} \right) = \\ &= \frac{\partial}{\partial x_j} \left(\underbrace{\overline{\rho \tilde{u}_i \tilde{u}_j}}_{\mathcal{L}_{ij}} + \underbrace{\overline{\rho \tilde{u}_i u_j''} - \overline{\rho \tilde{u}_i \tilde{u}_j}}_{\mathcal{C}_{ij}} + \underbrace{\overline{\rho u_i'' \tilde{u}_j}}_{\mathcal{R}_{ij}} + \underbrace{\overline{\rho u_i'' u_j''}}_{\mathcal{R}_{ij}} \right) \end{aligned} \quad (21)$$

Analogous for the convective term in the energy equation

$$\begin{aligned} \frac{\partial \overline{\rho e_0 u_j}}{\partial x_j} &= \frac{\partial}{\partial x_j} \left(\overline{\rho (\tilde{e}_0 + e_0'') (u_j + u_j'')} \right) = \frac{\partial}{\partial x_j} \left(\overline{\rho \tilde{e}_0 \tilde{u}_j} + \overline{\rho \tilde{e}_0 u_j''} + \overline{\rho e_0'' \tilde{u}_j} + \overline{\rho e_0'' u_j''} \right) = \\ &= \frac{\partial}{\partial x_j} \left(\underbrace{\overline{\rho \tilde{e}_0 \tilde{u}_j}}_{\mathcal{L}_{ij}} + \underbrace{\overline{\rho \tilde{e}_0 u_j''} - \overline{\rho \tilde{e}_0 \tilde{u}_j}}_{\mathcal{C}_{ij}} + \underbrace{\overline{\rho e_0'' \tilde{u}_j}}_{\mathcal{R}_{ij}} + \underbrace{\overline{\rho e_0'' u_j''}}_{\mathcal{R}_{ij}} \right) \end{aligned} \quad (22)$$

The terms denoted $\mathcal{L}_{ij}$, $\mathcal{C}_{ij}$, and $\mathcal{R}_{ij}$ in Eqs. 21–22 are often referred to as the Leonard stress, cross term, and Reynolds subgrid-scale stress, respectively. In the present work the Leonard stresses have not been evaluated explicitly. Instead a flux correction term has been introduced, later referred to as the Leonard flux correction term. The effect of the flux correction operation corresponds to the effect of the Leonard stress $\mathcal{L}_{ij}$ in the filtered equations. The reason for using this technique is that the finite-volume method used is considered to be tightly coupled to the LES filter. The filtered equations are not solved for explicitly, rather the numerical method acts as a box filter, which gives the filtered flowfield as a result.

The Leonard stresses as given in Eqs. 21–22 are not Galilean invariant, they are therefor often rewritten by combining the Leonard stress and the cross term on the form $\mathcal{L}_{ij} = \tilde{u}_i \tilde{u}_j - \tilde{u}_i \tilde{u}_j$, which is Galilean invariant.³⁴ However, since the numerical scheme is not Galilean invariant in itself, it is not believed to be of that great importance for this work to implement the stresses in the Galilean invariant formulation.

When evaluating the convective fluxes in the solver version without the flux correction technique implemented, the first terms on the right hand sides of Eqs. 21–22 are obtained using face averages of $\bar{\rho}$, $\tilde{u}_i$ and $\tilde{e}_0$. These face averages are obtained using the third-order upwind scheme described in Section IV-A. The Reynolds stress terms are approximated using the subgrid-scale model defined by Eqs. 5–7.

An approximation of the variation of the state vector on primitive form, i.e. $q = (\rho, u_i, p)$, over a cell face is be obtained as

$$q = q^A + \frac{\partial q}{\partial J} J + \frac{\partial q}{\partial K} K \quad (23)$$

where q^A is a cell face average obtained as the average of the two adjacent cells on each side of the cell face and $\frac{\partial q}{\partial J}$ and $\frac{\partial q}{\partial K}$ are, respectively, the derivatives of the state vector with respect to J and K , which are the two directions in computational space aligned with the cell face plane, see Figure (2). These derivatives are obtained as

$$\begin{aligned} \frac{\partial q}{\partial J} &= \frac{1}{2} (\tilde{q}_{i,j+1,k} - \tilde{q}_{i,j-1,k} + \tilde{q}_{i+1,j+1,k} - \tilde{q}_{i+1,j-1,k}) \\ \frac{\partial q}{\partial K} &= \frac{1}{2} (\tilde{q}_{i,j,k+1} - \tilde{q}_{i,j,k-1} + \tilde{q}_{i+1,j,k+1} - \tilde{q}_{i+1,j,k-1}) \end{aligned} \quad (24)$$

Using Eq. 23 to estimate the state vector values in Gauss points depicted in Figure (2), the Leonard

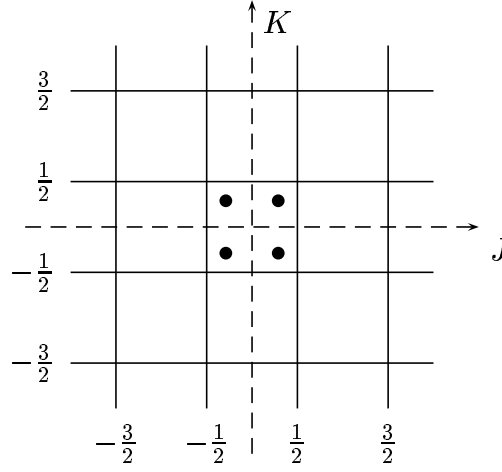


Figure 2. Leonard stresses are evaluated using integration based on Gauss points. The Gauss points are located at $(J = \pm \frac{1}{\sqrt{12}}, K = \pm \frac{1}{\sqrt{12}})$ and are indicated by black dots in the figure.

flux correction can be obtained as the difference between the integral based on Gauss points and the integral of face averages over the cell face according to

$$\begin{aligned} \text{Leonard Flux Correction} &= \int \int_{face} \rho q_p u_q n_q dS - \rho^A q_p^A u_q^A n_q S \\ &\simeq \sum_{G=1}^4 \rho^G q_p^G u_q^G n_q^G \Delta S - \rho^A q_p^A u_q^A n_q S \end{aligned} \quad (25)$$

where S and G denote cell face area and Gauss point, respectively.

D. Kirchhoff Surface Integral Formulation

Kirchhoff integration is a method for predicting the value of a property Φ governed by the wave equation, at a point outside a surface enclosing all generating structures.¹⁰ The method was originally used in the theory of diffraction of light and in other problems of an electromagnetic nature⁹ but has recently been extensively used for aeroacoustic applications. The integral relation is given by

$$\Phi(\mathbf{y}, t) = \frac{1}{4\pi} \int_S \left[\frac{\Phi}{r^2} \frac{\partial r}{\partial n} - \frac{1}{r} \frac{\partial \Phi}{\partial n} + \frac{1}{c_\infty r} \frac{\partial r}{\partial n} \frac{\partial \Phi}{\partial t} \right]_{\tau_r} dS(\mathbf{x}) \quad (26)$$

where $\mathbf{y}$ is a observer location in the far field and $\mathbf{x}$ a location on the surface. τ_r denotes that the expression within brackets is to be evaluated at retarded time, i.e. emission time. τ_r is related to the observer evaluation time, t , by the distance from the surface to the observer, $r = |\mathbf{y} - \mathbf{x}|$, and the speed of sound in the far-field region, c_∞ , as

$$\tau_r = t - \frac{r}{c_\infty} \quad (27)$$

The variable Φ that is to be evaluated in this case is the surface pressure. S denotes the surface enclosing all sound generating structures and n denotes the direction normal to the surface. Surface

S must be placed in a region where the flow is completely governed by a homogeneous linear wave equation with constant coefficients.¹¹ Greater detail on the Kirchhoff surface integration method is given in e.g. Freund *et al.*¹¹ and Lyrntzis.¹⁰

E. Computational Set-up

Figure (3) gives an schematic overview of the computational domain used for the LES reported in this paper. In this figure, the upper half of the three-dimensional domain is shown. In order to minimize the effect of reflections at the domain outlet on the predicted flowfield, a damping zone was added at the domain outlet. The functionality of this damping zone or buffer layer is described in more detail in Andersson *et al.*¹⁴ and Andersson.¹⁵ The axial extent of the physical part of the domain downstream of the nozzle exit is 2.5 meters, which is equal to 50 nozzle diameters ($D_j = 50 \text{ mm}$). The radial extent is $10 D_j$ at the nozzle-exit plane and $20 D_j$ at the domain outlet. The physical part of the computational domain was discretized using a block-structured boundary-fitted mesh with 33 mesh blocks and approximately 3×10^6 cells. Additionally 5×10^5 cells were used to discretize the damping zone. In order to avoid a center line singularity and to establish mesh homogeneity through out the domain, a combination of polar and Cartesian blocks was used. The grid cells are stretched in the downstream direction and radially towards the boundaries. The mesh has been described in more detail in Refs. 14–16. The nozzle geometry included in the calculation domain corresponds to the last contraction of the nozzle configuration in the experimental set-up.

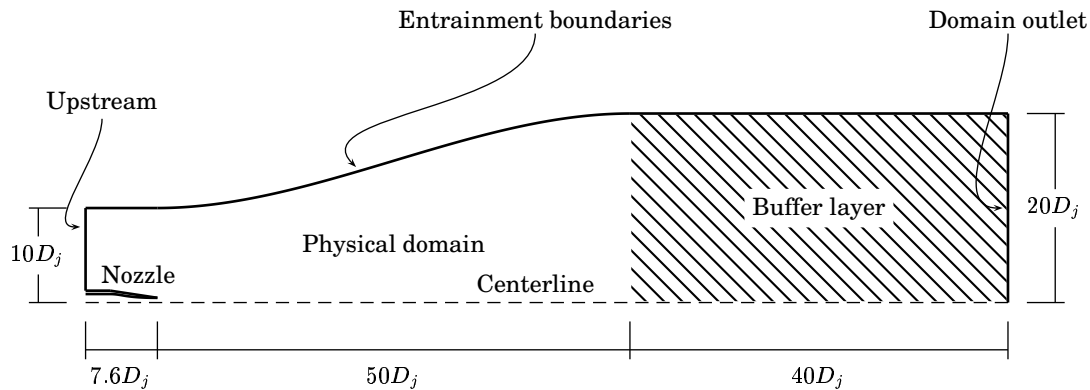


Figure 3. Computational set-up

For all free boundaries, i.e. the upstream and entrainment boundaries, absorbing boundary conditions based on the method of characteristics were adopted. Static pressure was specified at the domain outlet, i.e. at the downstream end of the buffer layer, and at the nozzle inlet P_0 and h_0 were specified. Constant values of these quantities were used in all simulation except in *Jet VII* where the total pressure was specified as an inlet profile $P_0 = P_0(r)$. In the *Jet III* simulation, synthesized turbulence fluctuations were added to the nozzle inlet velocity. The generation of these turbulence velocity components were described in more detail in Section IV-B. The velocities specified at the entrainment boundaries were interpolated from a corresponding RANS calculation. The calculation domain used for the RANS computation is sufficiently large to ensure that no disturbances caused by the outlet boundaries will reach the location of the LES domain boundaries. Figure (4) shows a comparison of the LES and RANS calculation domains.

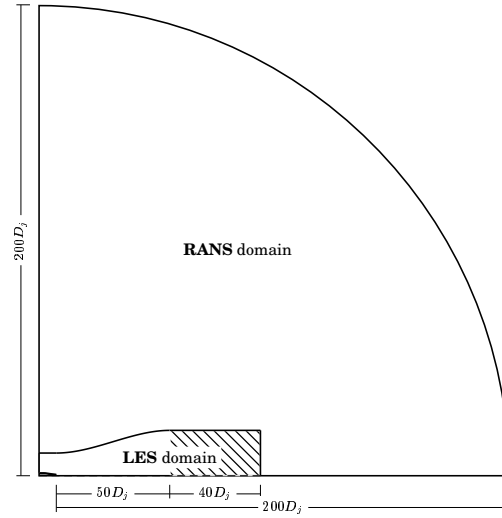


Figure 4. Entrainment velocities were obtained from RANS calculations performed using a significantly larger domain.

V. Results

A. Initial Jet Development

This section compares the results for the simulations mentioned in the introductory part of the paper, see Table (1). Comparisons of the simulations are made in terms of differences in shear layer development, flow statistics in the near-nozzle region and acoustic signature. The results of the simulations referred to as *Jet II* – *Jet VII* are compared to corresponding quantities obtained for the reference simulation, *Jet I*. For validation of the predicted flow quantities, measured data obtained for the same nozzle geometry and flow conditions have been included in the comparison of time-averaged flow quantities, far-field sound pressure levels and power spectra of far-field pressure.

1. Instantaneous Flowfield

Figure (5) shows entropy contours, $s = P/\rho^\gamma$, in the near-nozzle region for the reference case, *Jet I*, and in Figure (6) corresponding contours for *Jet II* – *Jet VII* are shown. Compared with the contours obtained for *Jet I*, the simulations in which the filter width has been modified are those that differs the most. Increasing the filter width leads, naturally since the subgrid-scale dissipation increases, to less structures in the initial part of the shear layer whereas decreasing the filter width gives a more rapid transition process and thicker shear layer close to the nozzle outlet, see Figures (6(d))–(6(e)). In *Jet II*, where the Reynolds number simulated is higher than in the reference simulation as well as in the other simulations reported in this work, structures appearing upstream of the nozzle exit close to the nozzle walls are much more prominent than in the other cases. These structures are, however, not fully trusted to be correctly represented since, as mentioned in the paper introduction, the mesh is the same as in the simulations with lower Reynolds number. The contours obtained for *Jet III*, *Jet IV* and *Jet VII* are similar to those obtained for *Jet I*.

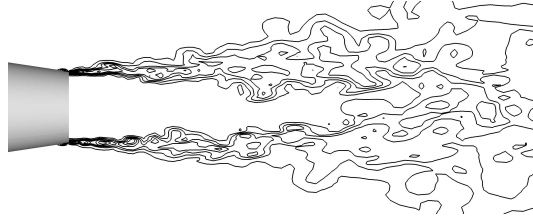
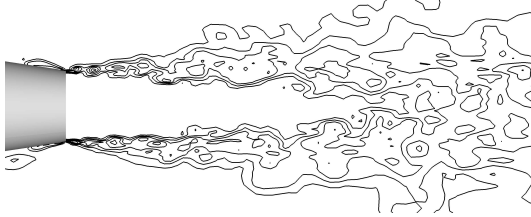


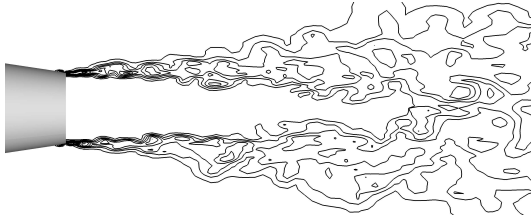
Figure 5. Entropy contours in the near-nozzle regions for the reference case (*Jet I*)



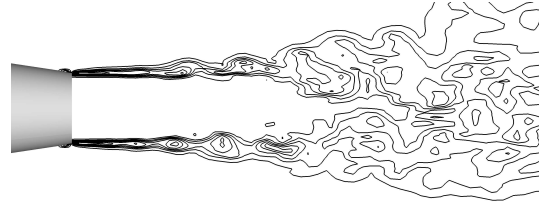
(a) *Jet II* ($Re_D = 9.0 \times 10^5$)



(b) *Jet III* (Synthesized inlet turbulence)



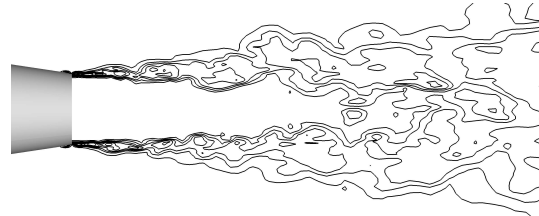
(c) *Jet IV* (Leonard flux correction)



(d) *Jet V* ($\Delta = \sqrt{2}(\Delta_1 \Delta_2 \Delta_3)^{1/3}$)



(e) *Jet VI* ($\Delta = \min(\Delta_1, \Delta_2, \Delta_3)$)



(f) *Jet VII* ($P_0 = P_0(r)$)

Figure 6. Entropy contours in the near-nozzle region for *Jet I* – *Jet VI*

2. Time-averaged Flowfield

In this section profiles of time-averaged quantities obtained in the initial jet region are compared for the different jet simulations. The predicted data are compared to measured data. Figure (7) shows locations of lines along which axial and radial profiles of time-averaged flow quantities have been

extracted. Note that for the radial profiles, flow quantities have been averaged both in time and in the azimuthal direction so as to improve the statistical convergence. Figures (8–11) show comparisons of

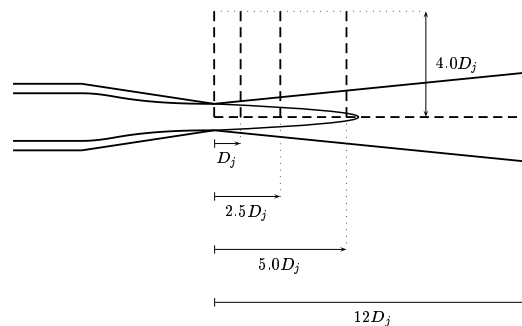


Figure 7. Profiles of axial velocity and second-order moments were obtained along the centerline and along four radial lines.

profiles of time-averaged flow data for *Jet I – Jet VII*. Note that the radial profiles have been staggered corresponding to the downstream location and that the turbulence intensities and $u'v'$ -correlations for these profiles have been multiplied by factors 8 and 140, respectively.

As can be seen from the Figures, there are only small differences in flow statistics in the initial flow region for most simulations made. The length of the predicted potential core is insignificantly changed from the reference simulation except for in the simulation with increased filter width (*Jet V*), in which the potential core is somewhat shorter, see Figure (8(a)). The decay rate of centerline axial velocity is also different for *Jet V*. Figure (9) shows a comparison of radial profiles of the $u'v'$ correlation, which is an indication of the degree of mixing in the jet. Again only small differences are observed for all simulations except for *Jet V*. For the profile extracted at $x/D_j = 1.0$, the $u'v'$ correlation obtained from *Jet V* is insignificantly larger than zero. For the subsequent locations, however, the levels in *Jet V* are higher than in the other simulations. This result is consistent with the observed shorter potential core in this case. The reason for this behavior is that the increased filter width results in a delay of the shear layer transition process as mentioned in the previous section. This in turn leads to that the transition process, once initiated, becomes more violent, which leads to higher levels of turbulence kinetic energy and a higher degree of mixing. The higher levels of turbulence kinetic energy in *Jet V* are also indicated by the increase in u_{rms} and v_{rms} seen in Figures (10) and (11), respectively. The predicted peaks in the centerline profiles of turbulence intensities obtained for *Jet V* are shifted towards the nozzle exit, see Figures (10(a)) and (11(a)). This is consistent with the fact that the potential core is shorter in this case since the location of maximum turbulence intensity coincides approximately with the location where the potential core ends. It should be noted, however, that the levels of turbulence intensities for *Jet V* are close to the peaks of the rms-values obtained for the measured data. However, turbulence intensities obtained in the measurements are significantly higher close to the nozzle exit than in the simulations, which might affect the maximum turbulence levels obtained. Some minor differences can be identified for the other simulations. *Jet II* and *Jet VI* gives slightly higher values of u_{rms} and $u'v'$ than most of the other simulations, which can be seen in the radial profiles depicted in Figures (10(b)) and (9). The centerline turbulence intensities are not fully converged, which makes the profiles uneven for axial locations downstream of the potential core region. Thus any conclusions drawn from differences observed in this region are rather uncertain. With this in mind it is noted that for *Jet VI*, the values of v_{rms} along the centerline is somewhat lower for axial locations downstream of $x/D_j = 4$. This is the simulation for which the subgrid-scale dissipation was decreased. Also, the levels of v_{rms} are higher for *Jet III* close to the nozzle exit. However, in this case one would expect even higher levels since synthesized turbulence has been added at the nozzle inlet.

This gives an indication of that a mesh refinement inside the nozzle might be required in order to avoid these fluctuations to be too affected by numerical dissipation.

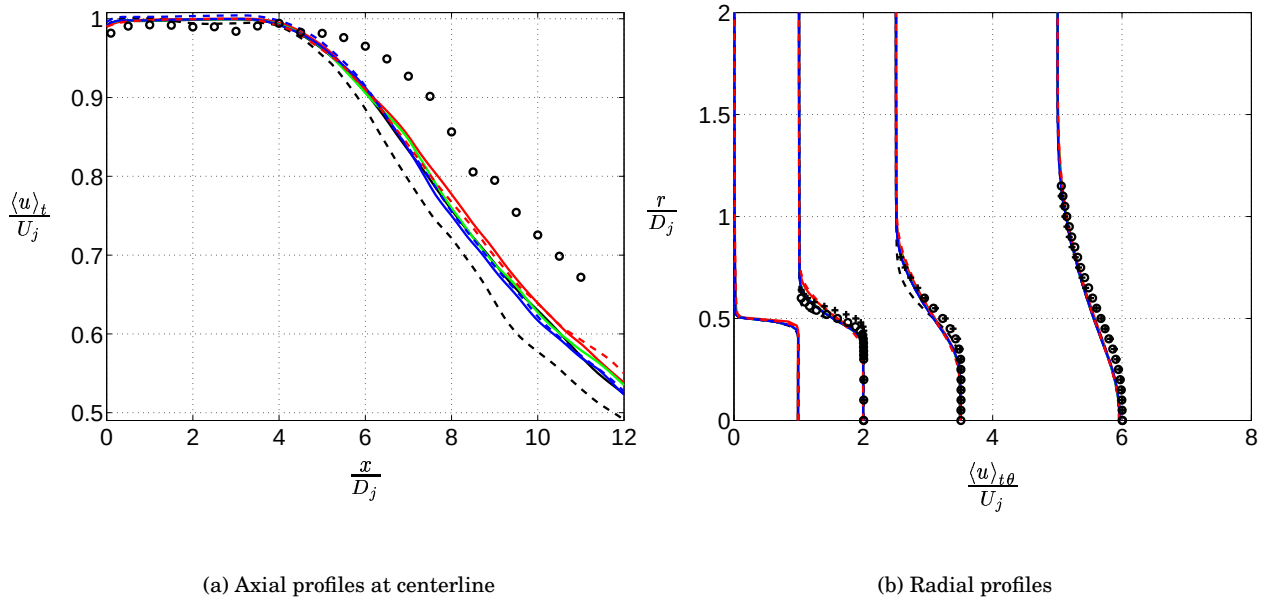


Figure 8. Profiles of axial velocity. Solid black lines denote *Jet I*; solid red lines denote *Jet II*; solid blue lines denote *Jet III*; solid green lines denote *Jet IV*; dashed black lines denote *Jet V*; dashed red lines denote *Jet VI* and dashed blue lines denote *Jet VII*. Black circles and pluses denote measured data^{21,24} obtained for the same nozzle geometry and flow conditions as the predictions.

B. Far-field Acoustic Signature

This section gives a presentation of sound pressure levels and power spectra of pressure signals for a few observer locations in the far field. The pressure signals were obtained using Kirchhoff surface integration. The observer locations coincide with the microphone locations in the experimental set-up,²² see figure (12). More detail on the method can be found in Refs. (14, 16).

Figure (13) depicts entropy contours in the jet region and dilatation contours further away from the jet for the *Jet I* simulation. In Figure (14) corresponding contour plots obtained for *Jet II – Jet VII* are depicted. As can be seen in all these figures, the sound waves radiated in the sideline direction are dissipated rather effectively. This is due to the fact that the mesh is not fine enough in that region to support the acoustic waves in the radial direction. This effect has been discussed in some detail in previously published work where the same computational set-up was used, see Refs. (14, 16). Comparing the dilatation contours obtained for *Jet I* with those obtained for *Jet II – Jet VII*, the most noticeable differences are found for *Jet V*, see Figure (14(d)). In this case the intensity of the high-frequency content in the sound radiated to the forward arc seems to be augmented. Moreover, the sound radiated to the rear arc is significantly changed. The introduction of the Leonard flux correction seems to improve the quality of the predicted radiated sound, see Figure (14(c)).

To further evaluate differences in the predicted acoustic signature, sound pressure levels and spectra of far-field pressure signals were obtained. The results are validated against available measured data provided by Laboratoire d'Etude Aérodynamiques. The results for *Jet I* have previously been

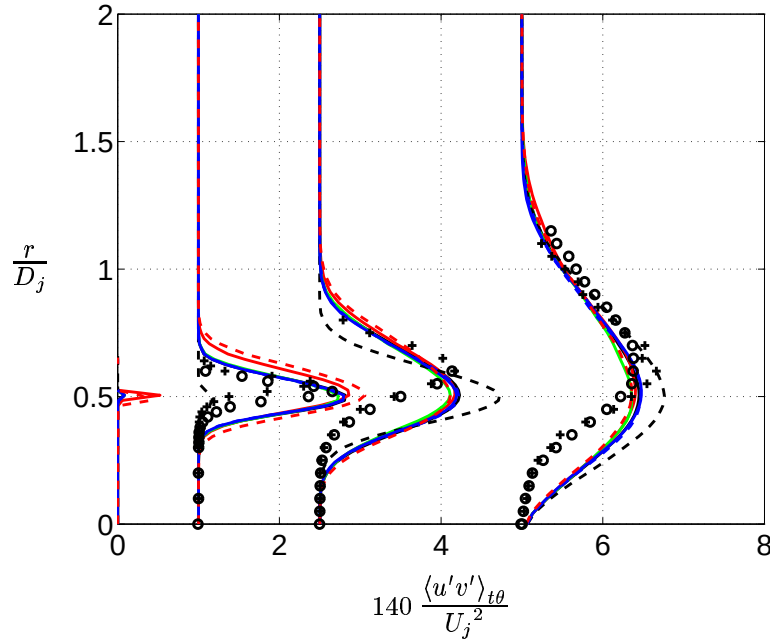
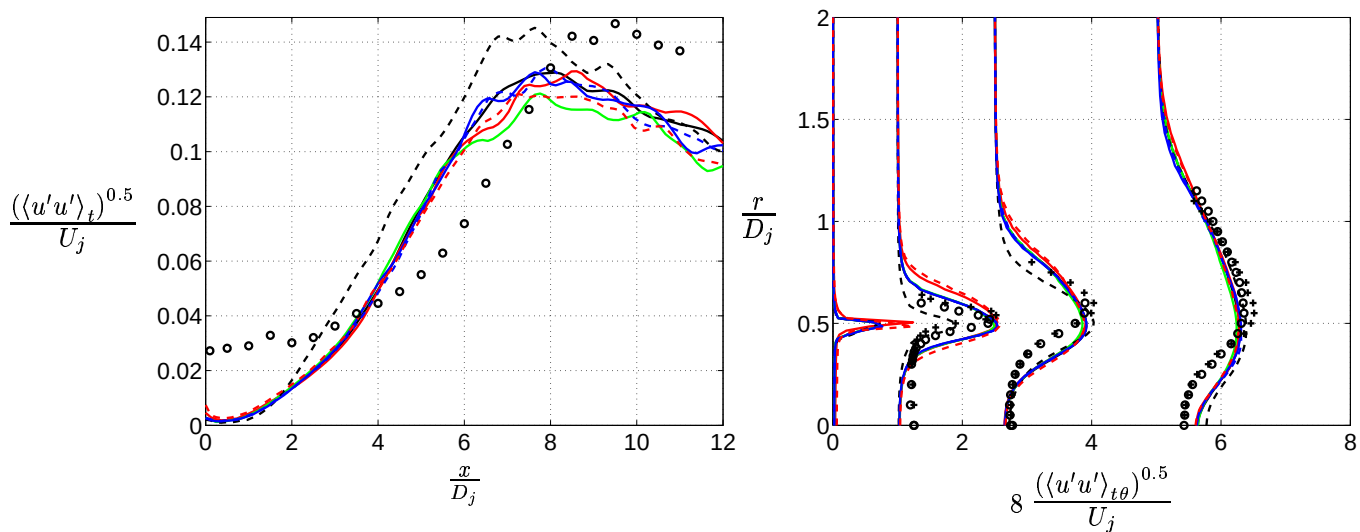


Figure 9. Radial profiles of $u'v'$. See also legend to Figure (8).

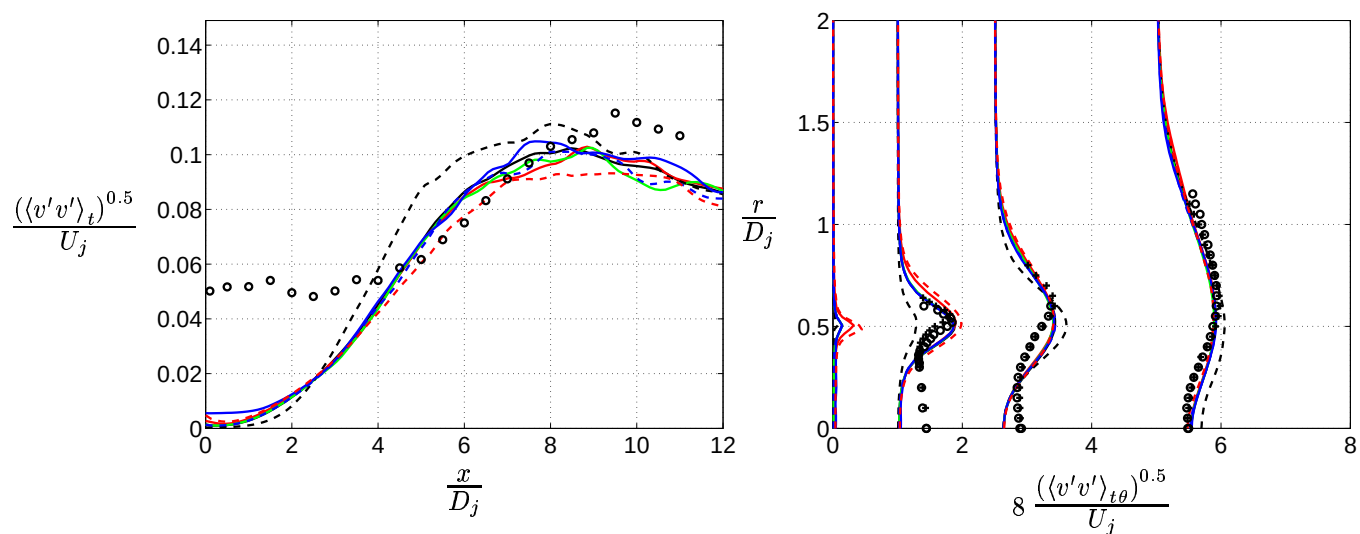
reported in Refs. (14, 16). Figures (15) through (20) give comparisons of predicted spectra and sound pressure levels. As can be seen in Figure (17(a)), the predicted spectra are better aligned with spectra obtained from measured data for *Jet IV* than for *Jet I*. Especially noticeable is that overprediction of high frequencies for 140° observer is decreased for *Jet IV*. It seems that introducing the Leonard flux correction technique improves the representation of the higher frequency content of the radiated sound and that this results in sound pressure levels in better agreement with measured levels for the forward arc observers, see Figure (17(b)). It should be noted, however, that for *Jet IV*, the sound pressure levels for the intermediate angles, i.e. the observers most affected by numerical dissipation, is even more underpredicted than for the reference simulation. The sound pressure levels obtained for *Jet V* are for all observers higher than in the reference simulation, see Figure (18(b)). The reason for this increase in SPL can be found in increased spectrum amplitudes. As can be seen in Figure (18(a)), the low frequency range amplitudes of the spectra are higher than those obtained for *Jet I* for all observers represented. As mentioned in the previous paragraph, this effect was also indicated in the dilatation contours for *Jet V*, see Figure (14(d)). The *Jet VI* simulation gives SPL somewhat lower for the rear arc observers than the reference simulation, see Figure (19(b)). *Jet II*, *Jet III* and *Jet VII* gives predicted sound pressure levels and spectra comparable to those of *Jet I*, see Figures (15), (16) and (20).



(a) Axial profiles at centerline

(b) Radial profiles

Figure 10. Profiles of u'_{rms} . See also legend to Figure (8).



(a) Axial profiles at centerline

(b) Radial profiles

Figure 11. Profiles of v'_{rms} . See also legend to Figure (8).

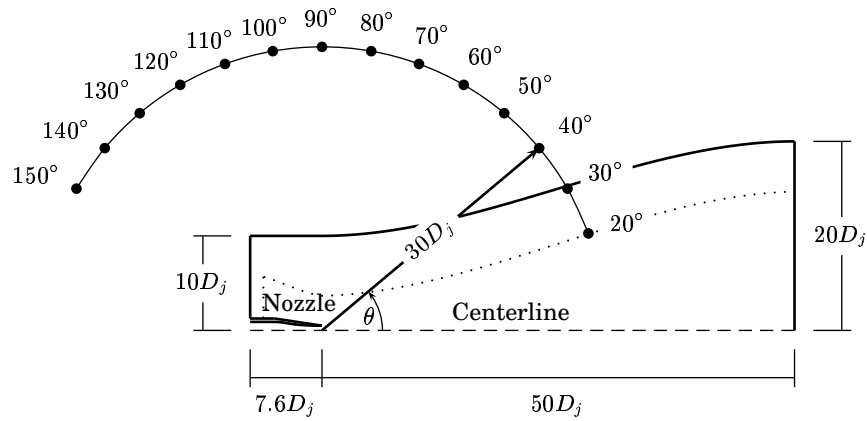


Figure 12. Acoustic pressure fluctuations were evaluated in 14 observer locations situated on an arc in the far-field region of the jet. The arc has its center at the nozzle-exit plane. The dotted line denotes the Kirchhoff surface used.

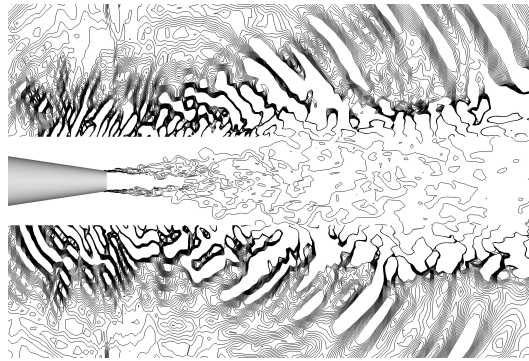


Figure 13. Entropy and dilatation contours for the reference case (*Jet I*)

VI. Concluding Remarks

In the present work, large-eddy simulation has been used for the simulation of a single-stream nozzle/jet configuration. The sensitivity of the flow in the near-nozzle region and the radiated sound to changes in subgrid-scale quantities, inflow conditions and Reynolds number have been investigated. The results have been compared to previously published results for the single-stream jet^{14,16} and measurements^{21,22,23} obtained for the same geometry and flow conditions. Short descriptions of the simulations for which results are reported in this paper are listed below.

Jet I	reference simulation ^{14, 16}
Jet II	The simulated Reynolds number corresponds to the Reynolds number for which the measurements were obtained
Jet III	synthesized inlet turbulence
Jet IV	Leonard flux correction
Jet V	increased filter width
Jet VI	decreased filter width
Jet VII	specified inlet profile

Only the simulation with increased filter width gave significant changes in the near-nozzle flow statistics. The increased subgrid-scale dissipation results in a lower degree of mixing close to the nozzle than for the other simulations presented. This in turn leads to a more violent mixing process once initiated, which can be seen in the higher levels of resolved turbulence intensities and overpredicted $u'v'$ correlation for this jet. The potential core is somewhat shorter in this simulation than in the others, which is consistent with the higher degree of mixing. Moreover, the rate of decay of the centerline axial velocity is slightly increased for this jet. None of the changes from the reference simulation gave a potential core length in better agreement with the measured data. It was discussed by Sagaut³⁵ that, although LES might be able to capture the physics of shear layer flows, quantities such as the potential core length in the jet case are almost never captured. In summary, it seems that, although some changes to the shear layer flow can be observed in the simulations made in this work, the flow statistics in the near-nozzle region is rather insensitive to changes in inflow parameters and subgrid-scale quantities.

A rough comparison of the predicted acoustic signatures was made by comparing contours of dilatation in the far field. Compared to the reference simulation, increasing the subgrid-scale filter width results in augmented high-frequency content of the forward radiated sound as well as significant changes to the sound radiated to the rear arc. The introduction of the Leonard flux correction terms seems to improve the quality of the predicted acoustic signature. The other simulations gives dilatation contours comparable to those obtained for the reference case.

Sound pressure levels in the far field were evaluated with a hybrid approach. Kirchhoff surface integration was utilized for the propagation of sound to locations in the far field. Introducing flux correction terms that approximate the effects of the Leonard stresses appearing in the spatially filtered governing equations gave a better representation of the high-frequency content of the radiated sound. Observer locations situated on the forward arc are affected to a higher degree than the rear arc observers by high-frequency noise generated in the shear layer region. Thus the improved representation of the higher frequency content results in sound pressure levels that are better aligned with the measured levels for the upstream observers. It should be mentioned, however, that this improvement comes with a penalty in terms of calculation cost. The iteration wall time when the Leonard flux correction was implemented in the solver was roughly 4/3 times the iteration time for the other simulations.

VII. Acknowledgment

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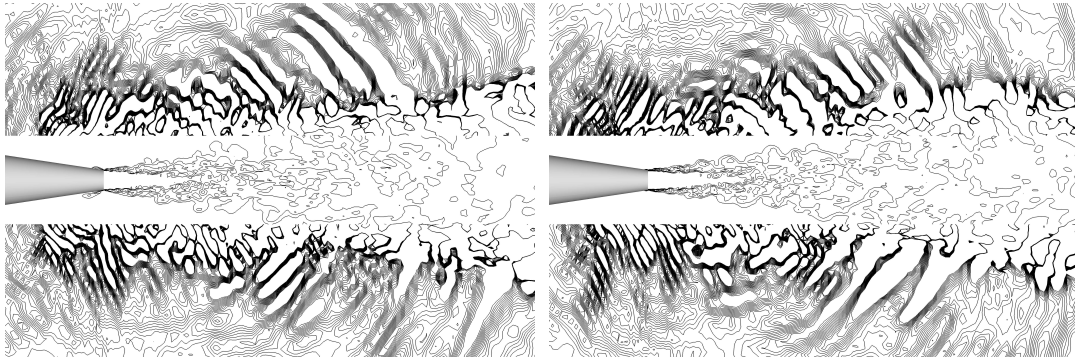
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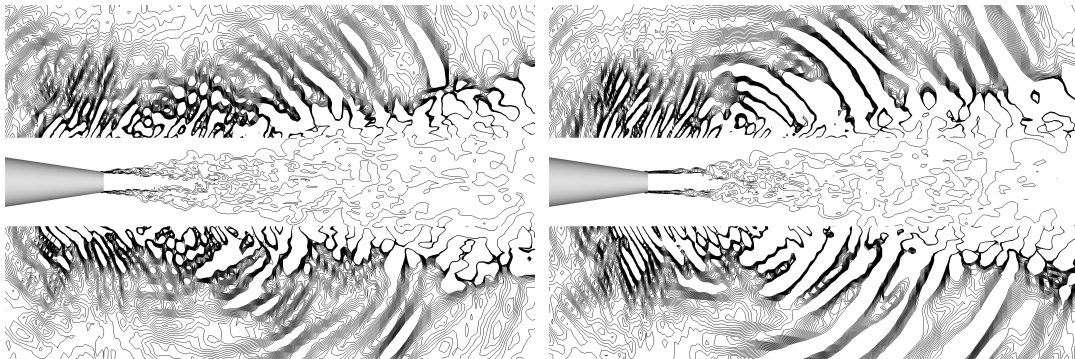
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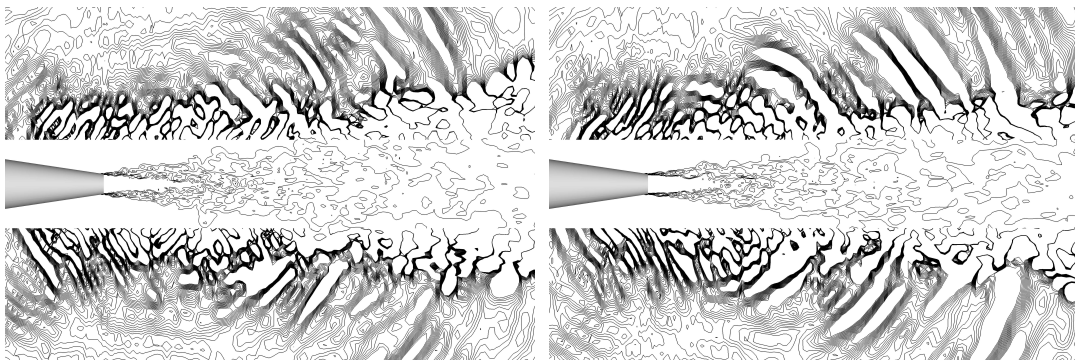
(a) *Jet II* ($Re_D = 9.0 \times 10^5$)

(b) *Jet III* (*Synthesized inlet turbulence*)



(c) *Jet IV* (*Leonard flux correction*)

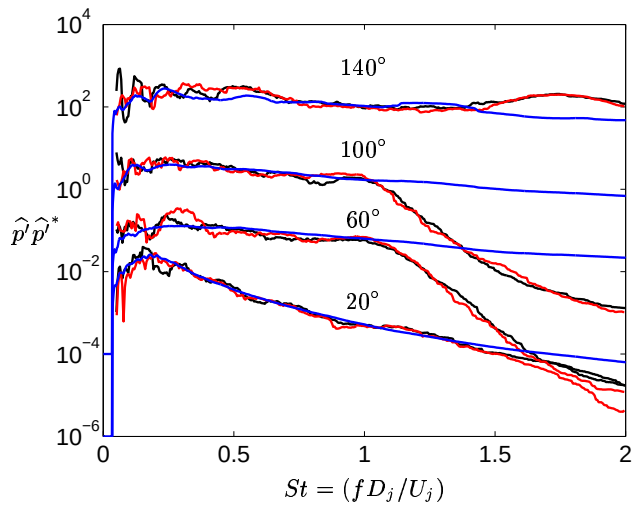
(d) *Jet V* ($\Delta = \sqrt{2}(\Delta_1 \Delta_2 \Delta_3)^{1/3}$)



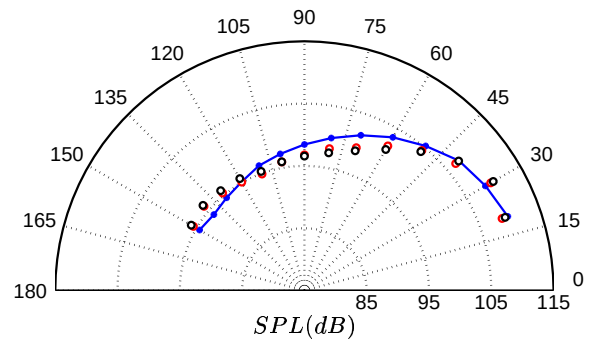
(e) *Jet VI* ($\Delta = \min(\Delta_1, \Delta_2, \Delta_3)$)

(f) *Jet VII* ($P_0 = P_0(r)$)

Figure 14. Instantaneous contours of dilatation and entropy for *Jet II* – *Jet VII*

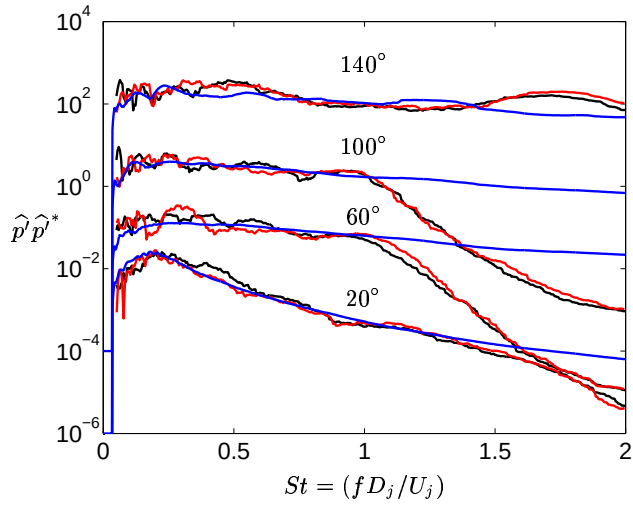


(a) Power spectra (*Jet II*)

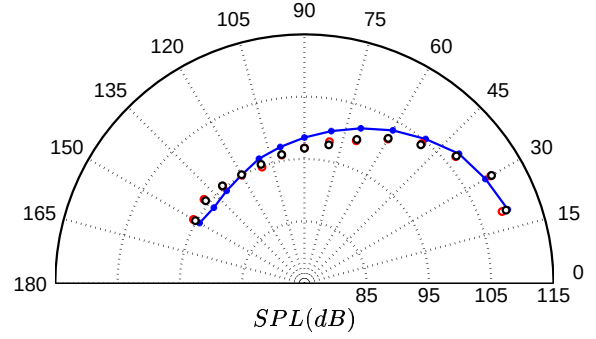


(b) Sound pressure levels (*Jet II*)

Figure 15. Power spectra and far-field sound pressure levels for *Jet II* ($Re_D = 9.0 \times 10^5$). The spectra have been staggered by multiplying the amplitude by a factor 10^{2n} , where $n = \left(\frac{\theta-20}{40}\right)$, θ being the angle from the jet axis. $\hat{p}'$ denotes the Fourier transform of the pressure fluctuation and $\hat{p}'^*$ its conjugate. Blue lines and circles denote measured data, red lines and circles correspond to the reference case *Jet I* and black lines and circles denote the predictions for *Jet II*.

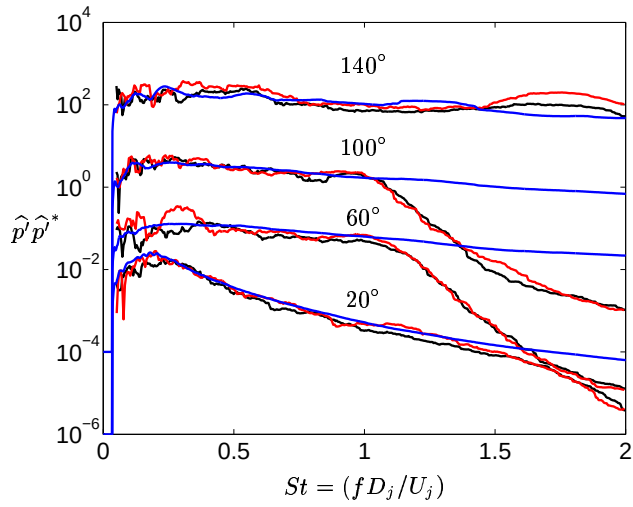


(a) Power spectra (*Jet III*)

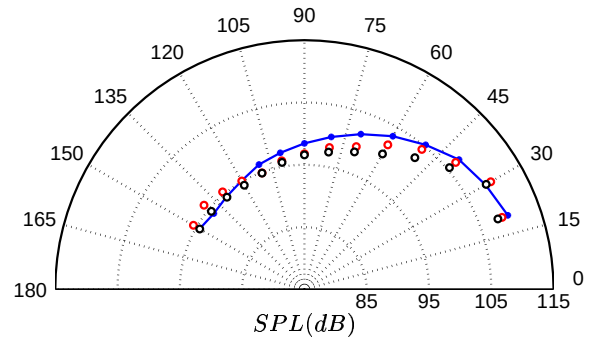


(b) Sound pressure levels (*Jet III*)

Figure 16. Power spectra and far-field sound pressure levels for *Jet III* (Synthesized inlet turbulence). For legend see Figure (15).

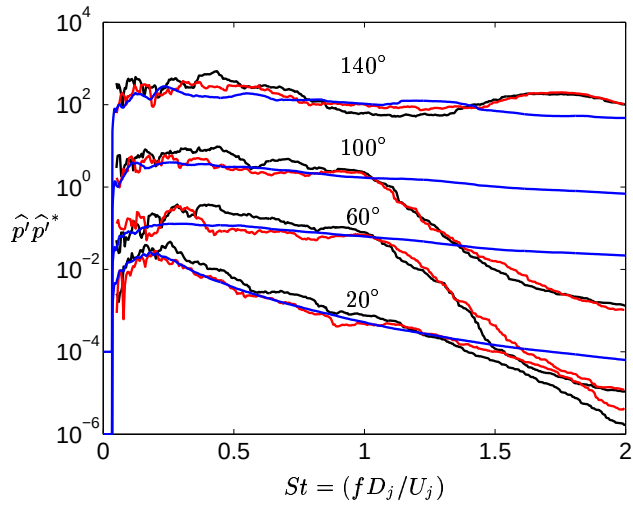


(a) Power spectra (*Jet IV*)

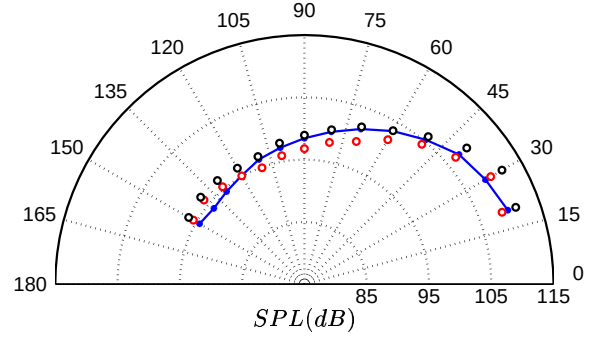


(b) Sound pressure levels (*Jet IV*)

Figure 17. Power spectra and far-field sound pressure levels for *Jet IV* (Leonard flux correction). For legend see Figure (15).

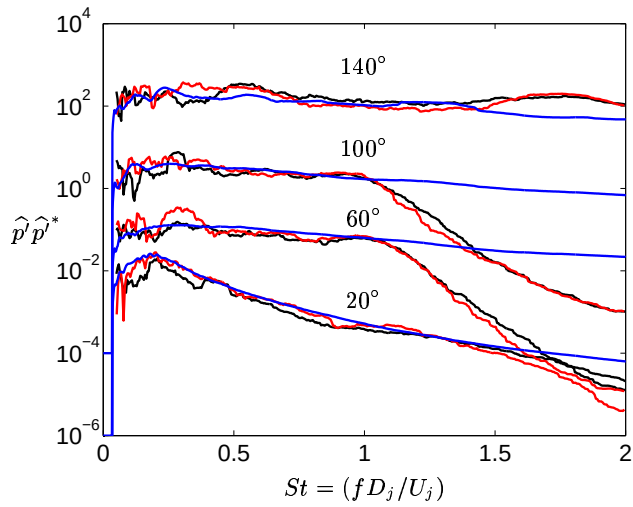


(a) Power spectra (*Jet V*)

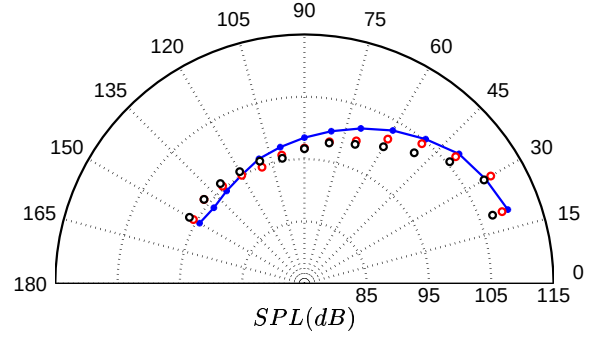


(b) Sound pressure levels (*Jet V*)

Figure 18. Power spectra and far-field sound pressure levels for *Jet V* ($\Delta = \sqrt{2}(\Delta_1\Delta_2\Delta_3)^{1/3}$). For legend see Figure (15).

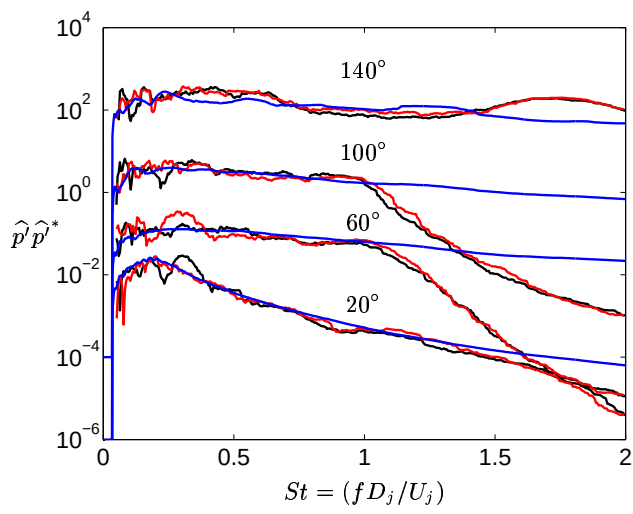


(a) Power spectra (*Jet VI*)

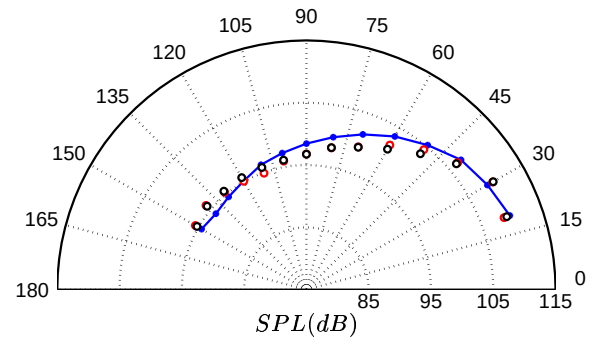


(b) Sound pressure levels (*Jet VI*)

Figure 19. Power spectra and far-field sound pressure levels for *Jet VI* ($\Delta = \min(\Delta_1, \Delta_2, \Delta_3)$). For legend see Figure (15).



(a) Power spectra (*Jet VII*)



(b) Sound pressure levels (*Jet VII*)

Figure 20. Power spectra and far-field sound pressure levels for *Jet VII* ($P_0 = P_0(r)$). For legend see Figure (15).

Paper IV

LES Prediction of Flow and Acoustic Field of a Coaxial Jet

By

N. Andersson, L.-E. Eriksson and L. Davidson

In proceedings for the 11th AIAA/CEAS Aeroacoustics Conference, AIAA 2005-2884, May 23-25, Monterey, California, 2005.

LES Prediction of Flow and Acoustic Field of a Coaxial Jet

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A compressible high-subsonic coaxial jet has been simulated using large-eddy simulation (LES). The acoustic field was extended to the far field using Kirchhoff surface integration. The jet Mach number based on the local speed of sound is approximately 0.9 for both the primary and secondary stream. The static temperature in the primary stream is three times that of the secondary stream.

In order to resolve the acoustic field, it is desirable to have a computational domain with a rather large radial extent and a mesh that is relatively fine even in the far-field regions. Furthermore, the mesh should be as equidistant as possible so as to minimize the introduction of numerical errors. In order to keep the number of cells down, the computational domain was divided into three regions: a well resolved near-wall LES region, a medium-resolution LES region optimized for propagation of acoustic waves, and a coarse LES region. Over the interfaces between these regions, the number of cells is increased by factor two in each direction. Special treatment of the interfaces between the regions is utilized in order to minimize undesirable numerical errors.

The radial extent of the computational domain increases downstream such that the flow in the outer boundary region can be assumed to be irrotational and axisymmetric. Hence, the flow outside the three-dimensional computational domain can be represented by a less expensive two-dimensional axisymmetric calculation. The interface between the full 3D LES region and the 2D region is based on azimuthally averaged quantities and acts as an absorbing boundary condition.

The Favre-filtered Navier-Stokes equations were solved using a finite-volume method solver with a low-dissipation third-order upwind scheme for the convective fluxes, a second-order centered difference approach for the viscous fluxes and a three-stage second-order Runge-Kutta technique in time. The computational domain was discretized using a block-structured boundary-fitted mesh with approximately 2.0×10^7 nodes. The calculations were performed on a parallel computer, using message-passing interface (MPI). A compressible form of Smagorinsky's subgrid-scale model was used to compute the subgrid-scale stresses.

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Nomenclature

Latin symbols

c	speed of sound
C_p	specific heat at constant pressure
C_R, C_I	Smagorinsky model coefficients
D	nozzle diameter
e	energy
k	kinetic energy
M	Mach number
p	pressure
Pr	Prandtl number
Q	state vector
q_j	energy diffusion vector
$\mathcal{R}$	correlation
r	radial coordinate or distance from source to observer
Re_D	Reynolds number
S_{ij}	strain rate tensor
T	temperature
t	time
U	velocity
u_i	Cartesian components of velocity vector
$\mathbf{x}$	flow location
x_i	Cartesian coordinate vector component
$\mathbf{y}$	far-field location

Greek symbols

Δ	filter width
δ_{ij}	Kronecker delta
γ	gas constant
μ	dynamic viscosity
ρ	density
σ_{ij}	viscous stress tensor
τ	temporal separation
τ_{ij}	subgrid-scale stress tensor
τ_r	retarded time
ξ	spatial separation

Subscripts

∞	ambient condition
j	jet-exit condition
p	primary stream
rms	root-mean-square
s	secondary stream
t	turbulent quantity

Superscripts

$-$	spatially filtered quantity
$'$	resolved fluctuation
$\sim$	spatially Favre-filtered quantity

$\wedge$	Fourier transform
SGS	subgrid scale

Symbols

$\langle \dots \rangle_\theta$	circumferentially averaged quantity
$\langle \dots \rangle_t$	time-averaged quantity

I. Introduction

STRICTER noise regulation for near-ground operations has made noise reduction in commercial aircraft a topic of growing interest in the aerospace industry. Despite progress in the development of CFD solvers, most of the noise prediction methods currently in use in industry are correlations based on empirical databases. The reason for this is the extreme demands for numerical accuracy in computational aeroacoustic (CAA) methods that place high demands on computational resources. However, to be able to account for changes in the flow by new noise reduction techniques or to predict differences in radiated sound in different engine concepts at an early design stage, more explicit approaches are required. With continuously increasing computer capacity and with the possibility to carry out parallel computations on PC clusters, computational aeroacoustics has now, to some extent, become feasible for industrial use.

Large-eddy simulation (LES) has recently been established as a numerical approach for the simulation of turbulent flows in situations in which detailed information on turbulence characteristics and flow physics is desirable and direct numerical simulation (DNS) is far too expensive. To be able to accurately predict flow-induced noise, flow simulation techniques of high accuracy have to be used. Using a compressible solver the acoustic pressure signal in far-field locations can be obtained directly from the flow simulation. This, however, leads to extremely expensive calculations since the mesh has to be fine enough through out the domain to ensure that acoustic waves are not damped by the numerical dissipation introduced by the numerical method. The calculation costs can be decreased by the use of a hybrid approach in which the hydrodynamic jet flowfield is obtained using DNS or LES and the far-field acoustics is obtained using an integral method, e.g. Kirchhoff¹ surface integration or the formulation by Ffowcs Williams & Hawkings,² see for example Freund *et al.*³ and Uzun *et al.*^{4,5} In a jet simulation the major part of the cells are often clustered in the hydrodynamic jet region so as to resolve turbulence structures in the jet. Thus using a hybrid approach like the ones mentioned above might, if the mesh is not as fine in the far field, give better results than a direct method since the cut-off frequency is higher in the integration surface region than in the far field.^{4,5} Andersson *et al.*^{6,7,8} used large-eddy simulation to study the flow and radiated sound of a subsonic (Mach 0.75) single-stream jet at different heating conditions. The acoustic field was extracted to the far field using Kirchhoff surface integration. The Reynolds number in that simulation was $Re_D = 5.0 \times 10^4$. Results were found to be in generally good agreement with measurements although the Reynolds number was significantly higher in the experiments. In the present work the LES/Kirchhoff approach has been applied to a coaxial jet configuration. The work has been done as part of the EU 6th framework programme CoJeN^a, an European effort to gain understanding of the physics behind noise generation in coaxial jets.

The objective of CoJeN is to develop and validate prediction tools that can be used by the aerospace industry to assess and optimize jet noise prediction techniques. The aim is to provide design tools that can be used to develop low noise nozzles for high bypass ratio (HBR) engines. For prediction techniques to be useful to industry, the methods must cope with realistic jet flows such as coaxial jet configurations with high velocities, significant velocity and temperature gradients, and arbitrary nozzle geometries. LES is currently not feasible for industrial design use because of long turnover times and a restriction to fairly simple geometries. However, the results of a detailed LES can be useful in the development of

^aComputation of Coaxial Jet Noise, contract number: AST3-CT-2003-502790

tools for industrial use. For example, higher-order statistics, such as two-point space-time correlations, can be evaluated.

LES and DNS have been used for jet flow applications in a number of investigations. These primarily study jets at moderate Reynolds number due to the high computational costs of making simulations of a high Reynolds number jet. Many of these studies have been carried out to predict jet noise. However, as jet flows are free shear flows frequently occurring in both nature and industrial applications, it is interesting to study the jet in itself. Some studies are thus pure investigations of flow phenomena.

The feasibility of using LES for both the flowfield and the radiated sound from a high-subsonic 6.5×10^4 Reynolds number jet has been discussed by Bogey *et al.*^{9,10,11} In Refs. (9, 10), the acoustic field was obtained directly from the flow simulation. Noise generation mechanisms were found to be relatively independent of the Reynolds number. In Ref. (11) Lighthill's¹² acoustic analogy was used in combination with compressible LES to obtain the acoustic field. Bogey & Bailly¹³ investigated the effects of inflow conditions on the flowfield and the radiated sound of a high Reynolds number, $Re_D = 4.0 \times 10^5$, Mach 0.9 jet. Both the flow development and the emitted sound were shown to depend appreciably on inlet parameters. Bogey & Bailly¹⁴ investigated the effects of Mach and Reynolds number on subsonic jet noise. Bodony *et al.*¹⁵ investigated the changes in jet mean field and far-field sound when simulating jets with different temperatures and Mach numbers. Results were found to be in agreement with published data.¹⁶ Freund¹⁷ used DNS to investigate sources of sound in a Mach 0.9 jet at a Reynolds number of $Re_D = 3.6 \times 10^3$. In this work the part of the Lighthill source that may radiate to the far field was isolated using Fourier methods. It was found that the peak of the radiating source coincides neither with the peak of the total source nor the peak of turbulence kinetic energy. Shur *et al.*¹⁸ made simulations of a cold Mach 0.9 jet at a Reynolds number of 1.0×10^4 . Radiated sound was successfully predicted using Ffowcs Williams & Hawkings² surface integral formulation. The simulation was made using only 5.0×10^5 cells. This work was done with the monotone-integrated LES (MILES) approach where the subgrid-scale model is essentially replaced by the numerical dissipation of the numerical method. Zhao *et al.*¹⁹ made an LES of a Mach 0.9 jet at Reynolds number 3.6×10^3 and a jet at Mach 0.4 and a Reynolds number of 5.0×10^3 . In this study, radiated sound was obtained both directly from the LES and by using Kirchhoff surface integration. The effect on the radiated sound of the subgrid-scale model was investigated. It was found that using a mixed subgrid-scale model resulted in both higher turbulence levels and sound levels. Uzun *et al.*^{4,5} made LES of a high-subsonic (Mach 0.9) single-stream jet at a Reynolds number of 4.0×10^5 . Sound pressure levels obtained using surface integral methods, e.g. Kirchhoff and Ffowcs Williams & Hawkings, were compared to results obtained using a volume integral method (Lighthill's acoustic analogy). The results obtained using the surface approaches were found to be comparable with the results obtained using Lighthill integration.

II. Overview of the Present Study

A high-subsonic coaxial jet with heated primary stream is studied. The static temperature in the primary stream (core flow) is roughly three times that of the secondary stream (bypass flow), leading to large temperature gradients in the shear layer between the two streams. The jet Mach number, $M_j = U_j/c_j$, is approximately 0.9 for both streams. The jet is expanded into stagnant surroundings and the temperature of the secondary stream is equal to the ambient temperature. Table 1 gives more detail on the flow properties for the simulated coaxial jet. In Figure (1), the nozzle geometry included in the computational domain is depicted: a (3/4) cowl nozzle with a center body.

No experimental data for validation of the LES predictions are available at this stage in the project. At a later stage, single-point multi-component statistics and spatial correlations will be available for validation of the predicted flow data, and the predicted acoustic signature will be evaluated using far-field acoustic measurements.

Table 1. Flow properties

<i>Jet flow conditions (primary (p) and secondary flow (s))</i>			
U_p	ms^{-1}	480.7	Fully-expanded jet velocity in the primary stream
U_s	ms^{-1}	306.8	Fully-expanded jet velocity in the secondary stream
M_p		0.861	Mach number (U_j/c_j) in the primary stream
M_s		0.902	Mach number (U_j/c_j) in the secondary stream
T_p	K	775.6	Static temperature in primary stream
T_s	K	288.14	Static temperature in secondary stream
T_{0p}	K	890.6	Total temperature in primary stream
T_{0s}	K	335.0	Total temperature in secondary stream
<i>Ambient conditions</i>			
P_∞	Pa	101325	
ρ_∞	kgm^{-3}	1.22556	
c_∞	ms^{-1}	340.174	
U_∞	ms^{-1}	0.0	
T_∞	K	288.14	
<i>Nozzle geometry</i>			
D_p	m	0.137	
D_s	m	0.275	

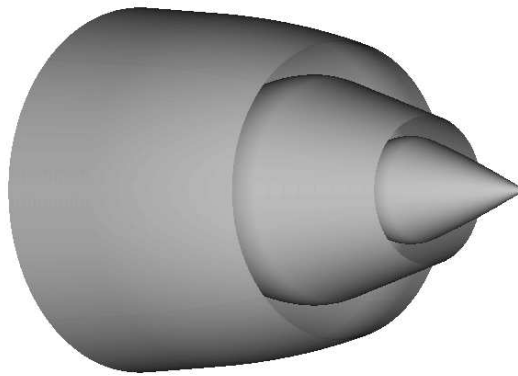


Figure 1. The CoJeN short cowl nozzle

III. Governing equations

The equations solved are the spatially Favre-filtered continuity, momentum and energy equations,

$$\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial (\bar{\rho} \tilde{u}_i)}{\partial x_i} = 0 \quad (1)$$

$$\frac{\partial (\bar{\rho} \tilde{u}_i)}{\partial t} + \frac{\partial (\bar{\rho} \tilde{u}_i \tilde{u}_j)}{\partial x_j} = -\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial \bar{\sigma}_{ij}}{\partial x_j} + \frac{\partial \tau_{ij}}{\partial x_j} \quad (2)$$

$$\begin{aligned} \frac{\partial (\bar{\rho} \tilde{e}_0)}{\partial t} + \frac{\partial (\bar{\rho} \tilde{e}_0 \tilde{u}_j)}{\partial x_j} = & -\frac{\partial \bar{p} \tilde{u}_j}{\partial x_j} + \\ & + \frac{\partial}{\partial x_j} \left(C_p \frac{\mu}{Pr} \frac{\partial \tilde{T}}{\partial x_j} + q_j \right) + \frac{\partial}{\partial x_j} (\tilde{u}_i (\bar{\sigma}_{ij} + \tau_{ij})) \end{aligned} \quad (3)$$

where $\bar{\sigma}_{ij}$ and τ_{ij} are the Favre-filtered viscous stress tensor and subgrid-scale viscous stress tensor, respectively. These are here defined as

$$\bar{\sigma}_{ij} = \mu \left(2\tilde{S}_{ij} - \frac{2}{3}\tilde{S}_{mm}\delta_{ij} \right) \quad (4)$$

$$\tau_{ij} = \mu_t \left(2\tilde{S}_{ij} - \frac{2}{3}\tilde{S}_{mm}\delta_{ij} \right) - \frac{2}{3}\bar{\rho}k^{SGS}\delta_{ij} \quad (5)$$

where k^{SGS} is the subgrid-scale kinetic energy

$$k^{SGS} = C_I \Delta^2 \tilde{S}_{mn} \tilde{S}_{mn} \quad (6)$$

μ_t the subgrid-scale dynamic viscosity

$$\mu_t = C_R \bar{\rho} \Delta^2 \sqrt{\tilde{S}_{mn} \tilde{S}_{mn}} \quad (7)$$

and $\tilde{S}_{ij}$ is the Favre-filtered strain rate tensor given by,

$$\tilde{S}_{ij} = \frac{1}{2} \left(\frac{\partial \tilde{u}_i}{\partial x_j} + \frac{\partial \tilde{u}_j}{\partial x_i} \right) \quad (8)$$

The subgrid heat flux appearing in the Favre-filtered energy equation is modeled using a temperature gradient approach

$$q_j = C_p \frac{\mu_t}{Pr_t} \frac{\partial \tilde{T}}{\partial x_j} \quad (9)$$

The filter-width in Eqs. (6) and (7) is a representative local grid cell width. In the present work both $\Delta = (\Delta_1 \Delta_2 \Delta_3)^{1/3}$ and $\Delta = \min(\Delta_1, \Delta_2, \Delta_3)$ have been used. The subgrid-scale model used in this work is the Smagorinsky part of the model proposed by Erlebacher *et al.*²⁰ for compressible flows. Constants C_R and C_I that appear in Eqs. (6) and (7) are the Smagorinsky model constants, of which the latter is a compressibility correction constant. These are here given by

$$\begin{cases} C_R &= 0.012 \\ C_I &= 0.0066 \end{cases} \quad (10)$$

The system of governing equations, Eqs. (1–3), is closed by making assumptions on the thermodynamics of the gas considered. It is assumed that the gas is thermally perfect, i.e. it obeys the gas law. Furthermore, the gas is assumed to be calorically perfect, which implies that internal energy and enthalpy are linear functions of temperature.

IV. Sound Propagation

Sir James Lighthill's description of his acoustic analogy,¹² i.e. the analogy between the full non-linear flow and the linear theory of acoustics, is often referred to as the start of aeroacoustics. Combining the time derivative of the continuity equation and the divergence of the momentum equation, the flow equations may be rewritten in a wave operator form.¹² An analytical solution may be obtained by the use of a free-space Green's function and thus acoustic pressure signals in far-field observer locations can be estimated by integration over a volume containing all sound-generating sources. It is important to note that no interaction between the propagating sound wave and the flowfield in which it is propagating is explicitly considered when the integral form of the analogy is used, i.e. no refraction or convection effects are taken into account. The equation produced by this pioneer work has later been modified in several ways to include, for example, the effects of the mean-flow acoustic interactions, flow inhomogeneities and surfaces.²¹

A. Kirchhoff Surface Integral Formulation

Kirchhoff integration is a method for predicting the value of a property, Φ , governed by the wave equation, at a point outside a surface enclosing all generating structures.²² The method was originally used in the theory of diffraction of light and in other problems of an electromagnetic nature¹ but has recently been extensively used for aeroacoustic applications. The integral relation is given by

$$\Phi(\mathbf{y}, t) = \frac{1}{4\pi} \int_S \left[\frac{\Phi}{r^2} \frac{\partial r}{\partial n} - \frac{1}{r} \frac{\partial \Phi}{\partial n} + \frac{1}{c_\infty r} \frac{\partial r}{\partial n} \frac{\partial \Phi}{\partial t} \right]_{\tau_r} dS(\mathbf{x}) \quad (11)$$

where $\mathbf{y}$ is a observer location in the far field and $\mathbf{x}$ a location on the surface. τ_r denotes that the expression within brackets is to be evaluated at a retarded time, i.e. emission time. τ_r is related to the observer evaluation time, t , by the distance from the surface to the observer, $r = |\mathbf{y} - \mathbf{x}|$, and the speed of sound in the far-field region, c_∞ , as

$$\tau_r = t - \frac{r}{c_\infty} \quad (12)$$

The variable Φ that is to be evaluated in this case is the surface pressure. S denotes the surface enclosing all sound generating structures and n denotes the direction normal to the surface. Surface S must be placed in a region where the flow is completely governed by a homogeneous linear wave equation with constant coefficients.³ Greater detail on the Kirchhoff surface integration method is given in e.g. Freund *et al.*³ and Lyrantzis.²²

The strength of the sources in the hydrodynamic jet region decays slowly downstream, which means that the downstream end of a closed surface will be located in regions of considerable hydrodynamic fluctuations. It is thus common practice to use Kirchhoff surfaces that are not closed in the upstream and downstream ends. Freund *et al.*³ showed that the errors introduced by using such surfaces are small if the main portion of the sound sources are within the axial extent of the surface and if lines connecting observer locations with locations in the hydrodynamic region, representing the main sources of sound, intersect the surface. Rahier *et al.*²³ found that the downstream closing surface makes only a minor contribution to the radiated sound. In the work reported in this paper a Kirchhoff surface open in both the upstream end and the downstream end is used, see Figure (2).

The Kirchhoff surface integral method is less computationally expensive and storage demanding than a method based on volume integrals, e.g. Lighthill's acoustic analogy, since it is only surface data that must be stored and evaluated. Moreover, the integrand includes only first-order derivatives, while the integrand in Lighthill's acoustic analogy contains second-order derivatives in time or space. One of the main strengths of the Kirchhoff surface integral approach, contrary to many other methods, is that as long as all sources of sound, flow inhomogeneities and objects that in some way affect the radiated sound are bounded by the surface, all these effects are taken into account. This means, for

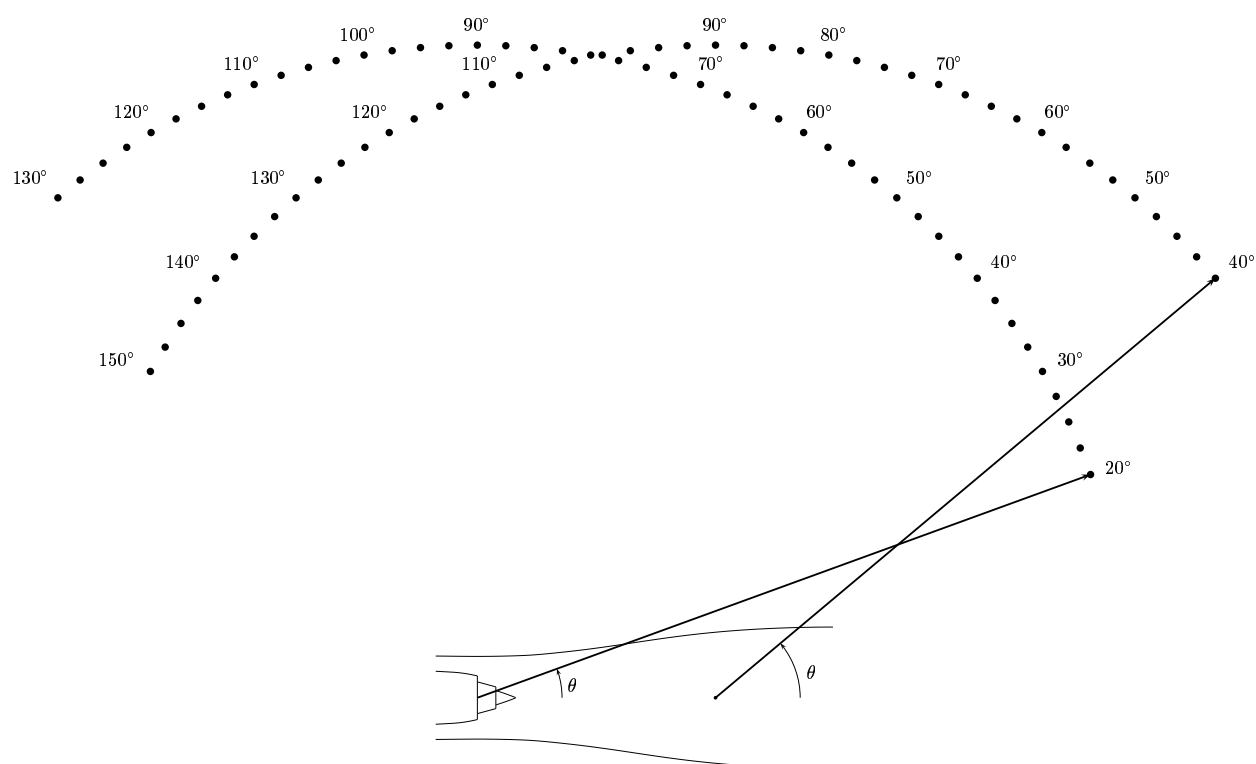


Figure 2. Kirchhoff integration surface and far-field observer locations are depicted. In total 90 observer locations are represented; 45 observers are situated on an arc centered on the centerline at the exit plane of the secondary nozzle and 45 observers on an arc centered at a location 1.5m downstream of the first location. The latter location is approximately where the potential core ends in RANS computations made for the same case.

example, that no extra treatment is needed to be able to predict the effects of refraction and convection. This is one of the attractive features of this method. Moreover, both a strength and a weakness of the surface integral approach is the effect of numerical dissipation on the predicted sound. Acoustic waves approaching the integration surface will be low-pass filtered by the numerical method. A result of this artificial dissipation is that spurious high-frequency noise not supported by the mesh and scheme will be filtered out and thus not contribute to the predicted far-field sound. On the other hand, the numerical dissipation affects the ability to capture the high-frequency range of the acoustic pressure spectra, and thus the quality of the prediction depends strongly on the mesh resolution in the Kirchhoff surface region. The ideal situation would be to place the integration surface as far as possible from the sources to ensure that all non-linearities appear within the integration surface. However, the distance from the source to the surface is often limited by the numerical dissipation introduced by the method. Using Ffowcs William & Hawkings² surface integral method on a porous surface formulation allows the surface to enter regions with weak hydrodynamic fluctuations, which means that the surface can be placed somewhat closer to the sources of sound, see e.g. Shur *et al.*,¹⁸ Rahier *et al.*²³ and Uzun *et al.*^{4,5} However, if placed in the linear acoustic field the Kirchhoff surface formulation and the formulation of Ffowcs William & Hawkings gives identical results. Uzun *et al.*^{4,5} found the results obtained using the two surface integration methods to be similar for integration surfaces placed rather close to the jet.

V. Method

A. Numerical Scheme

The Favre-filtered Navier-Stokes equations were solved using a finite-volume method solver with a third-order low-dissipation upwind scheme for the convective fluxes and a centered difference approach for the diffusive fluxes. The temporal derivatives were estimated using a second-order three-stage Runge-Kutta technique. The solver used is based on the G3D family of codes developed by Eriksson.²⁴ The convective scheme is a combination of centered and upwind biased components that have been used with good results for LES of free shear flows by Mårtensson *et al.*²⁵ and, more recently, for shock/shear-layer interaction by Wollblad *et al.*²⁶ The coefficients of low-dissipation upwind scheme for estimation of the convective flux over a cell face are derived using a third-order polynomial $Q(x)$ to represent the variation of the flow state in the direction normal to the face as

$$Q(x) = A + Bx + Cx^2 + Dx^3 \quad (13)$$

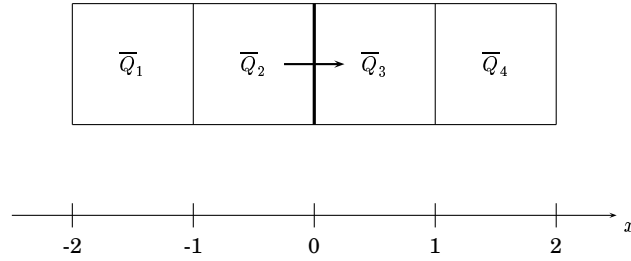


Figure 3. The face state, Q_0 , is estimated using the state in four neighboring cells.

The face state is evaluated using the interpolated value $Q(0)$, modified to include upwinding by adding the third derivative of $Q(x)$ according to

$$\begin{aligned} Q_0 &= Q(0) + \delta Q'''(0) = A + 6\delta D = \\ &= C_1 \bar{Q}_1 + C_2 \bar{Q}_2 + C_3 \bar{Q}_3 + C_4 \bar{Q}_4 \end{aligned} \quad (14)$$

where coefficient δ in front of the upwind term has been chosen by numerical experiments²⁵ to be $(1/96)$ in order to introduce only a small amount of upwinding. The result is the low-dissipation third-order upwind scheme with the coefficients

$$\begin{cases} C_1 = - \left(\frac{1}{12} + \delta \right) = -\frac{9}{96} \\ C_2 = \left(\frac{7}{12} + 3\delta \right) = \frac{59}{96} \\ C_3 = \left(\frac{7}{12} - 3\delta \right) = \frac{53}{96} \\ C_4 = - \left(\frac{1}{12} - \delta \right) = -\frac{7}{96} \end{cases} \quad (15)$$

For more detail on the numerical scheme, see Eriksson²⁴ or Andersson.⁸

B. Computational Set-up

The computational domain is divided into three parts: a high-resolution LES region near the nozzle walls, a medium dense LES region and a coarse LES region. Figure (4) shows the location of these three mesh regions. The maximum size of the computational cells in the medium-resolution region is

chosen such that the mesh in this region will support acoustic waves of frequencies up to 15 kHz, using 8 cells per wavelength. In this region the cells are approximately equidistant. In the high-resolution LES region, the cells are twice as fine in each direction as in the medium dense region and the cells are stretched somewhat towards the nozzle walls. The cells in the coarse LES region are twice as wide in each direction as those in the medium-resolution LES region. The computational domain was discretized using a block-structured boundary-fitted mesh with 159 mesh blocks and approximately 2.0×10^7 nodes. Figure (5) depicts the mesh in the near wall fine LES region. The mesh is constructed using a combination of Cartesian and polar mesh blocks so as to ensure mesh homogeneity in the radial direction throughout the domain. A block with square cross section is used at the centerline to avoid a centerline singularity.

In order to minimize the effect of reflections at the domain outlet on the predicted flowfield, a damping zone was added at the domain outlet. The functionality of this damping zone, or buffer layer, is described in more detail in Andersson *et al.*⁶ and Andersson.⁷ At the nozzle inlet total pressure and total enthalpy are specified. Entrainment velocities at the coarse LES domain boundary are obtained by using a 2D extension of the domain representing the flow outside the LES domain, see Figure (4). This procedure will be discussed in more detail in Section D.

Although the flow is rather well resolved in the near wall regions, since the Reynolds number is very high, this must be considered as being a coarse LES. To be able to represent the near wall behavior of the flow wall functions are used where needed.

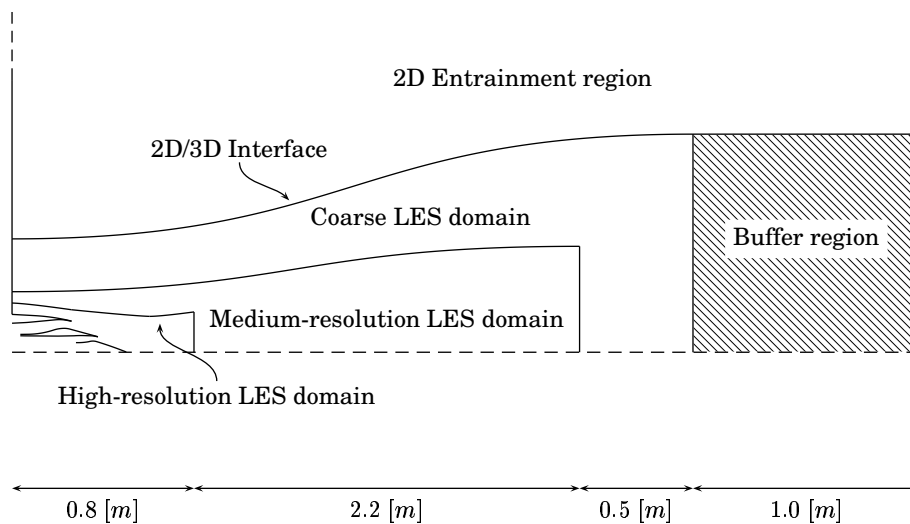
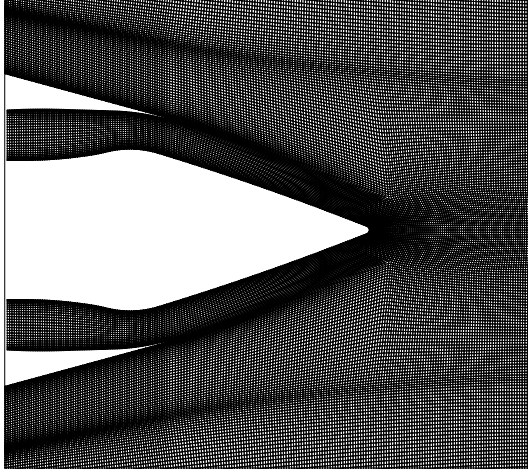


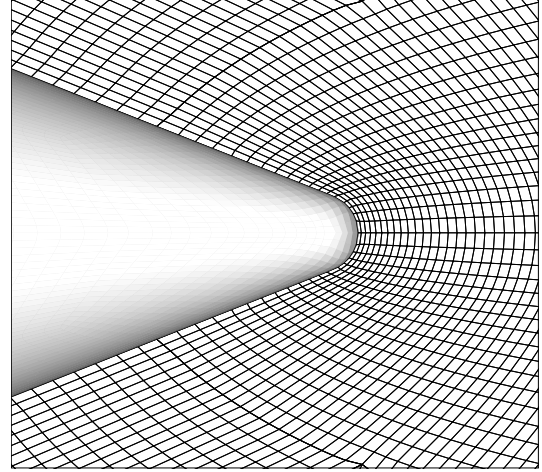
Figure 4. The computational domain is divided into three regions: a high-resolution region for resolution of boundary layers and initial shear layers, a medium LES region optimized for propagation of acoustic waves and a coarse LES region.

C. Stepwise Increased Resolution

To keep the numerical accuracy high and thus minimize the low pass filtering of the acoustic waves as they travel through the calculation domain, the mesh has to be rather fine even in the far-field regions.



(a) A slice through the calculation domain made at $z = 0$, i.e. a xy -plane, in the fine LES region is depicted. The figure shows the primary flow nozzle and the center body.



(b) A close up of the cone tip region is depicted. The mesh in this region is generated by mapping the x -plane surface of a square Cartesian block starting at the cone tip to the cone surface.

Figure 5. Mesh slices in the fine LES near-nozzle region

Moreover, it is desirable to have a mesh as equidistant as possible in order to minimize the introduction of numerical errors. The high resolution in the far field regions together with the extreme demands for resolution near the nozzle walls and in the shear layers results in that a very large number of cells have to be used to discretize the computational domain. To keep the number of cells down and at the same time being able to ensure high resolution near the nozzle walls and in the initial shear layer regions, the mesh used in this work has been refined in two distinct steps. In each of these steps the number of cells is increased by a factor two in each direction. In order to minimize the introduction of numerical errors, special treatment is utilized for estimation of the convective and diffusive fluxes over the interfaces for which this kind of refinement occurs, see Figure (6). The flux over the cell face one step in on the coarse side of the interface, i.e. at $x = -1$, is obtained using a corresponding coarse cell on the fine side, constructed as the average of the closest eight fine cells, see Figure (6b). For the block interface and the face one step in on the fine side ($x = 0.5$), cell reconstruction is used to generate corresponding fine cells on the coarse side of the interface, see Figure (6c). The fluxes over the interface are evaluated on the fine side and corresponding flux contributions with opposite signs are added to the cell on the coarse side.

In the cell reconstruction procedure it is assumed that the spatial variation of the state vector in one direction can be represented by a second-order polynomial, $Q(x) = A + Bx + Cx^2$. By integration of this polynomial over three cells the coefficients A , B and C , can be expressed as functions of the cell averages of the state vector according to

$$\begin{aligned} A &= \frac{1}{24} [-\bar{Q}_1 + 26\bar{Q}_2 - \bar{Q}_3] \\ B &= \frac{1}{2} [-\bar{Q}_1 + \bar{Q}_3] \\ C &= \frac{1}{2} [\bar{Q}_1 - 2\bar{Q}_2 + \bar{Q}_3] \end{aligned} \quad (16)$$

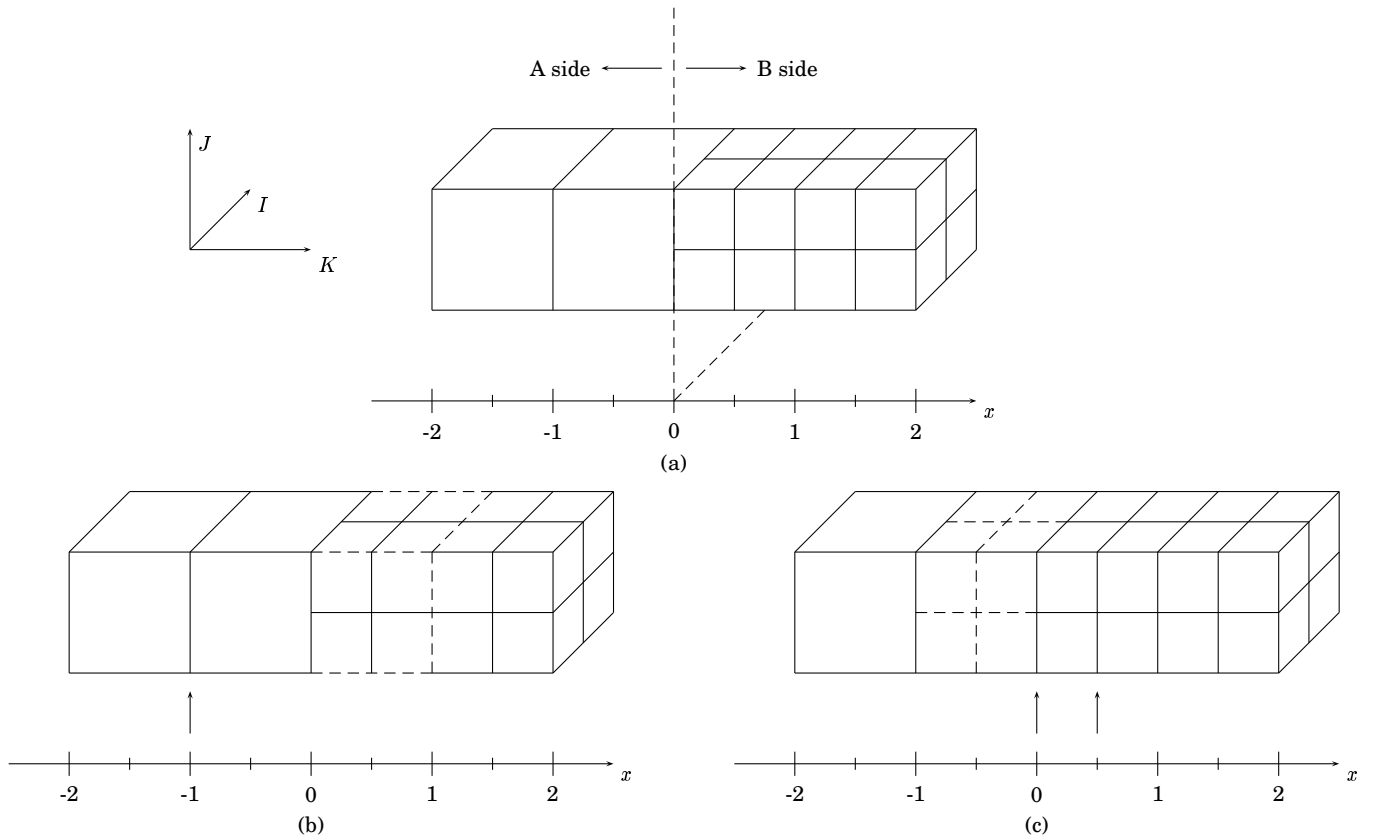


Figure 6. Over the interface between mesh block A and mesh block B, the number of cells is increased by a factor two in each direction (a). The fluxes one step in on the A side of the interface, i.e. at $x = -1$, are estimated by using a corresponding coarse cell on the fine side obtained by averaging over eight fine cells (b). The fluxes on the block interface and one step in on the B side, i.e. at $x = 0.5$, is estimated using cell reconstruction on the coarse side of the interface (c).

The cell average of the state vector for a reconstructed cell is calculated by integration according to

$$\bar{Q} = \frac{1}{x_2 - x_1} \int_{x_1}^{x_2} Q(x) dx \quad (17)$$

where x_1 and x_2 represents the boundary locations of the new cell. The cell reconstruction is made in two steps. In a first step, both of the cells on the coarse side are divided in the I and J directions into four cells, matching those on the fine side of the interface. In this operation only the neighboring cells on the coarse side of the interface are used, see Figure (7). As an example of the reconstruction procedure, the state vector for the cell in the upper right corner in Figure (7) is obtained as

$$\begin{aligned} \bar{Q}_{i,j,k}^{++} = & \frac{1}{64} [-\bar{Q}_{i-1,j+1,k} + \bar{Q}_{i-1,j-1,k} + \bar{Q}_{i+1,j+1,k} - \bar{Q}_{i+1,j-1,k}] + \\ & + \frac{1}{8} [-\bar{Q}_{i-1,j,k} + \bar{Q}_{i+1,j,k} - \bar{Q}_{i,j-1,k} + \bar{Q}_{i,j+1,k}] + \bar{Q}_{i,j,k} \end{aligned} \quad (18)$$

The new cells closest to the interface on the coarse side are now divided in the direction normal to the interface by cell reconstruction involving two layers of cells on each side of the interface. In this

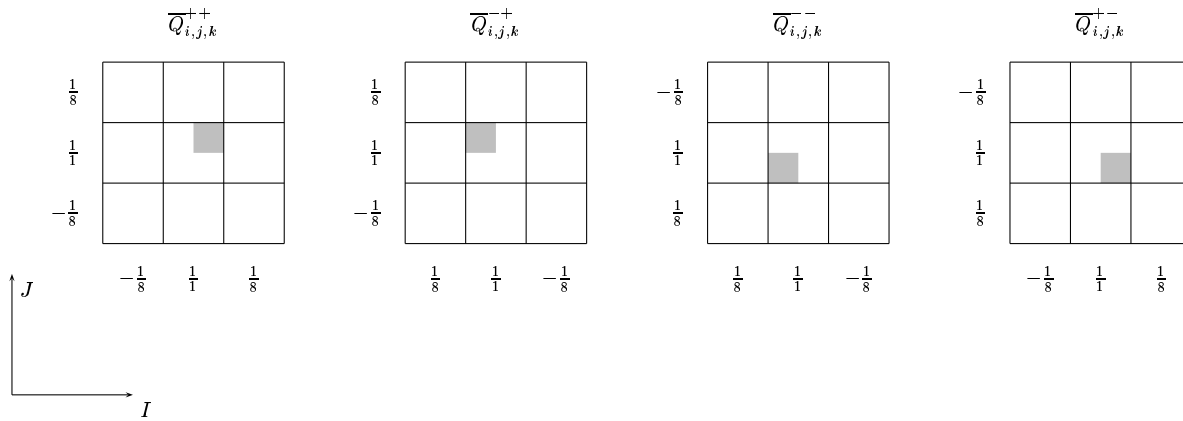


Figure 7. Cell reconstruction (step 1). Generate cells on the coarse side of the interface, matching the cells on the fine side. This step only involves cells on the coarse side.

reconstruction step, the average of the two B-side cells is used to represent the fine side of the interface, see Figure (8).

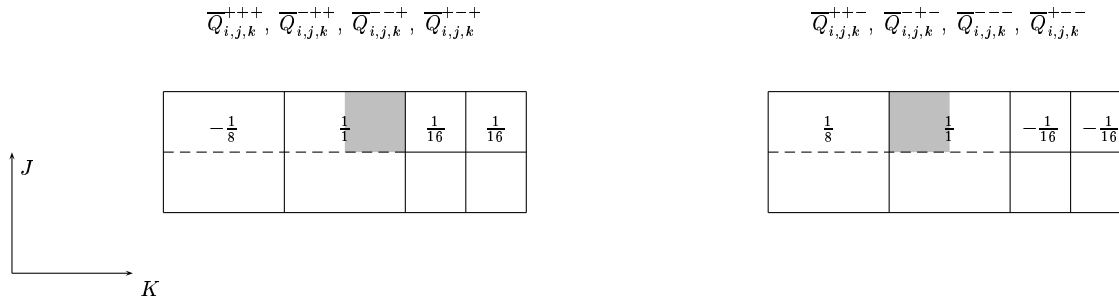


Figure 8. Cell reconstruction (step 2). The cells on the coarse side are reconstructed to match those on the fine side of the interface. In the reconstruction the average of two cells on the fine side are used to represent the fine side of the interface.

D. Entrainment Boundary Conditions

The radial extent of the computational domain increases downstream such that the flow in the domain boundary region can be assumed to be irrotational and axisymmetric. Hence, the flow outside the three-dimensional computational domain can be represented by a less expensive two-dimensional axisymmetric calculation, see Figure (4). The domain for this 2D calculation extends 4.5 meters radially from the jet centerline. To minimize boundary reflections, the 2D/3D interface acts as an absorbing boundary condition on both sides. The flow properties of the boundary conditions for the 3D domain are specified using flow information from the 2D domain, and the boundary condition for the 2D domain is defined using azimuthally averaged flow information from the 3D domain, see figure (9). The 2D domain has an extent in the tangential direction corresponding to the width of the scheme used for the convective fluxes, i.e. four cells. Azimuthal averages are obtained for the four cell rows adjacent to the interface highlighted in figure (9). Rotational periodic interfaces are used for the block faces in the tangential direction for the 2D domain. This entrainment boundary approach enables the use of a

rather narrow 3D domain. Furthermore, the outer boundaries of the 2D domain are located relatively far from the jet axis, which simplifies the definition of these boundary conditions.

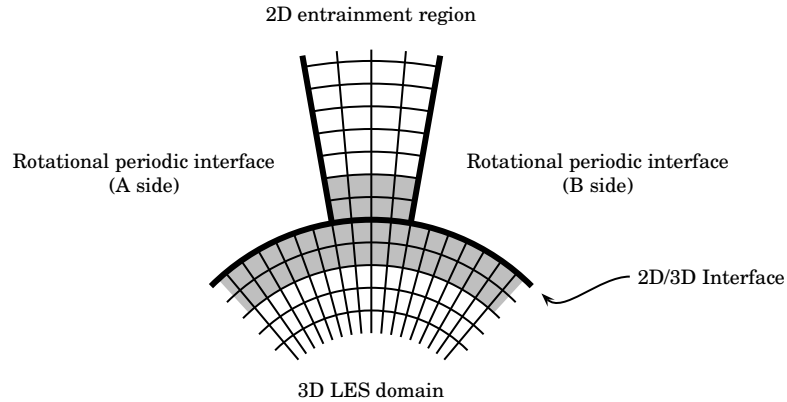


Figure 9. Interface between the 2D entrainment region and the 3D region. The interface acts as an absorbing boundary condition on both sides. The flow information specified at the interface boundary is obtained using azimuthally averaged flow information on both sides of the interface. The filled area denotes cells used to obtain these azimuthally averaged flow quantities.

VI. Results

In this section results for two simulations made for the coaxial nozzle configuration will be presented. These two simulations will hereafter be referred to as *case 1* and *case 2*, respectively. *Case 1* is the result of a first preliminary large-eddy simulation of the coaxial jet configuration. After evaluation of *case 1*, three changes were made to the simulation set-up for reasons that will be discussed in more detail later. The second case was simulated with these changes included and the effects on the predicted flowfield will be discussed in some detail in this section. The changes made were the following:

1. The SGS filter width was decreased from $\Delta = (\Delta_1 \Delta_2 \Delta_3)^{(1/3)}$ in *case 1* to $\Delta = \min(\Delta_1, \Delta_2, \Delta_3)$ in *case 2*.
2. When *case 1* was run, it was found that selective artificial damping for density was needed to ensure stability in the simulation. In *case 2* this artificial damping was replaced with a TVD limiter for density. This results in a less dissipative scheme and thus improves the numerical accuracy. As will be shown later this change had some positive effects on the shear layer flow.
3. In *case 1*, constant values of P_0 were specified at the nozzle inlet boundaries for both the core flow and the bypass flow whereas for *case 2* the inlet total pressures were given as profiles, $P_0 = P_0(r)$, which increased the boundary layer thickness in this simulation.

In the first simulation it was found that the mixing in the outer shear layer was very low close to the nozzle. Once the transition process was initiated, large ring shaped structures were generated, which eventually broke down into smaller structures. However, the low degree of mixing in the initial shear layer region results in that the ring shape of the vortex structures keeps their identity quite far downstream of the nozzle exit. This led to levels of turbulence kinetic energy significantly higher than what was obtained in RANS computations for the same nozzle configuration and flow conditions.

Although the lower levels of turbulence kinetic energy found in the RANS computations are not considered as a proof of that the LES results are unphysical, it gives an indication of that something might be wrong. It was found that the shear layer between the core flow and the bypass flow was very thin and stable. This was, unlike the previously mentioned observation, seen in the RANS results as well, which gives an indication of that this flow behavior might be correct. Nevertheless, this was the reason for changing the nozzle inlet boundary conditions and improve the accuracy of the numerical scheme by replacing the artificial damping with a TVD limiter. A velocity profile ought to give thicker boundary layers and hence a thicker shear layer. The observations for the *case 1* jet mentioned in this paragraph led to the changes of the simulation set-up listed above.

Although the initial jet region is well resolved, the aspect ratio of the cells in the nozzle regions is high. The cells are quite wide in the tangential direction as compared to the radial direction. Using $\Delta = (\Delta_1 \Delta_2 \Delta_3)^{(1/3)}$ as the filter width in the Smagorinsky subgrid-scale model leads to that the amount of dissipation introduced is not representative for the resolution in the radial direction. Several possible representations of the filter width for cells with high aspect ratio are described by Sagaut.²⁷ However, in this case the shortest side of each cell was chosen as a rough estimate. Decreasing the filter width to $\Delta = \min(\Delta_1, \Delta_2, \Delta_3)$ enhanced the mixing in the outer shear layer. The change from artificial damping to a TVD limiter further increased the degree of mixing in the outer shear layer. Moreover, this change led to a more unstable inner shear layer. A probably more physically correct behavior of the outer shear layer was established in the second simulation.

The time-average results presented in Section B and the two-point space-time correlations presented in Section C have been obtained from databases of samples for a subset of the computational domain using a sample rate of 100 [kHz]. 650 samples were obtained for *case 1* and 1500 samples for *case 2*. The time step used was chosen such that the CFL number did not exceed 0.5, which gave a time step of $\Delta t = 1.0 \times 10^{-7}$ [s], i. e. a sample rate of 100 [kHz] means that data are saved every hundred time step.

A. Instantaneous Flowfield

This section gives some examples of snap shots of the flow for *case 1* and *case 2*. Figures (10) and (11) show contours of density for *case 1* and *case 2*, respectively. From these figures, the differences in the two simulations regarding the initial mixing are quite visible. The initial part of the outer shear layer in the *case 1* simulation has no structures present at all whereas in *case 2*, small vortical structures are generated directly at the nozzle lip. As discussed previously, the lack of initial mixing in *case 1* leads to that the shape of vortical structures generated in the initial shear layer remain quite far downstream of the nozzle exit, which means that these structures grow quite large in size before they are broken down into smaller structures. The degree of mixing in the shear layer between the core flow and the bypass flow is generally very low. This is especially noticeable in the first simulation where this shear layer becomes more or less a surface separating the two streams. The vortical structures in the outer shear layer affect this surface in that the pairing processes occurring in the outer shear layer generate a wave-like behavior of the surface, see Figure (10). This finally leads to the collapse and break up of the shear layer/surface near the end of the inner potential core region. The inner shear layer seems to be rather stable in *case 2* as well but in this case small structures are generated in the initial part of the shear layer. The reason for the low degree of mixing in the inner shear layer is believed to be physically correct and caused by a combination of stabilizing factors. First of all, the velocity and momentum ratios are reversed, i.e. the velocity in the primary stream is higher than the velocity in the secondary stream, but, owing to temperature differences, the momentum flux is lower in the primary stream than in the secondary stream. This might have a stabilizing effect on the shear layer between the two streams. Moreover, the flow in the initial part of the inner shear layer is not fully axial but has a negative velocity component in the radial direction, which makes the shear layer concave. This curvature of the shear layer might also have a stabilizing effect on the flow. Comparing Figure (10)

with Figure (11), the structures generated once the inner shear layer breaks up is significantly larger in *case 1* than in *case 2*. This might be an effect of the large coherent structures that are present in the outer shear layer in the *case 1* jet.

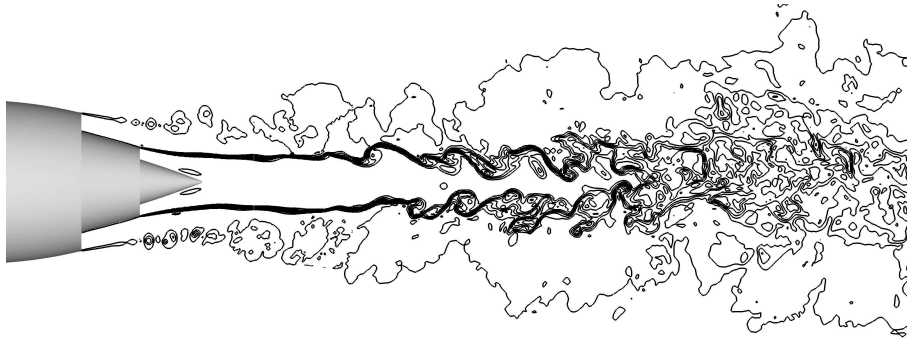


Figure 10. Instantaneous density contours (*case 1*)

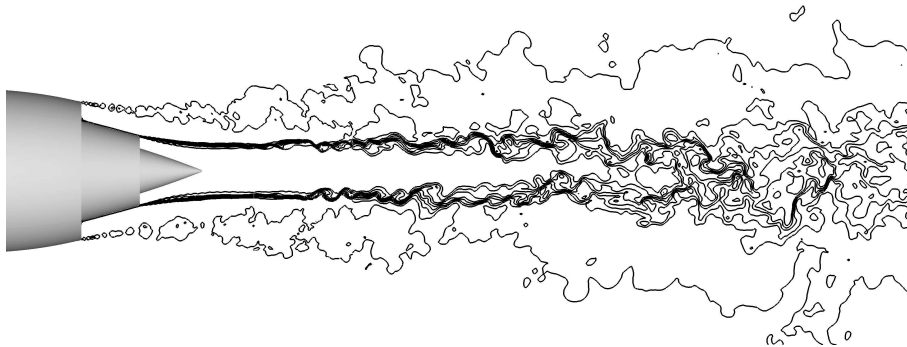


Figure 11. Instantaneous density contours (*case 2*)

Figures (12) and (13) show instantaneous contours of entropy, $s = P/\rho^\gamma$. Again the differences in initial mixing for the two jets are quite visible. Comparing the figures, it seems that the potential core is longer in the latter case, which might be explained by the fact that the mixing in *case 1*, once initiated, is more violent and efficiently breaks the potential core region down.

Figures (14(a)) and (14(b)) again show contours of entropy in the initial shear layer region. The thicker shear layers seen in *case 2* is probably an effect of that the inlet total pressures have been specified as profiles in this case.

In Figures (15–16) iso surfaces of density visualizing the flow structures in the inner and outer shear layers are shown. As can be seen in Figure (15), although the shear layers seems to be stable in both cases, the structures that occurs in the iso surfaces obtained for *case 1* are very different from those in the iso surfaces obtained for *case 2*. In *case 1* only large structures can be seen as was indicated in Figure (10). These large structures in the inner shear layer arise when vortical structures in the outer shear layer affect the inner shear layer. It is these large structures that eventually lead to the break-up of the inner shear layer in *case 1*. In *case 2*, Helmholtz instabilities in the form of ring shaped vortical structures can be seen in the initial part of the inner shear layer. As the shear layer develops, these vortices are deformed and axial structures are generated. This leads to that the inner shear

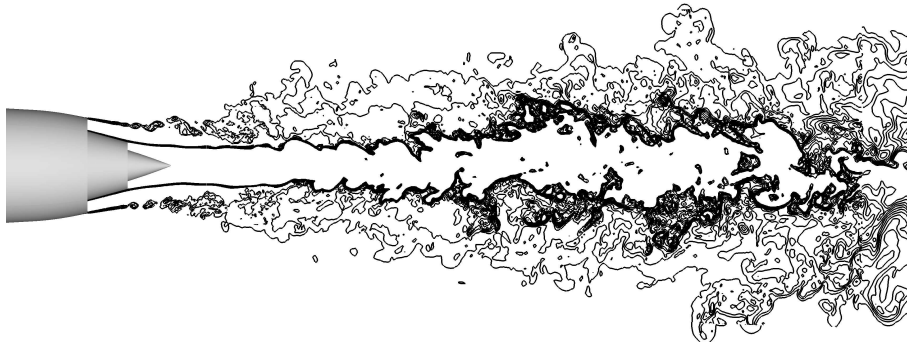


Figure 12. Instantaneous entropy contours (*case 1*)

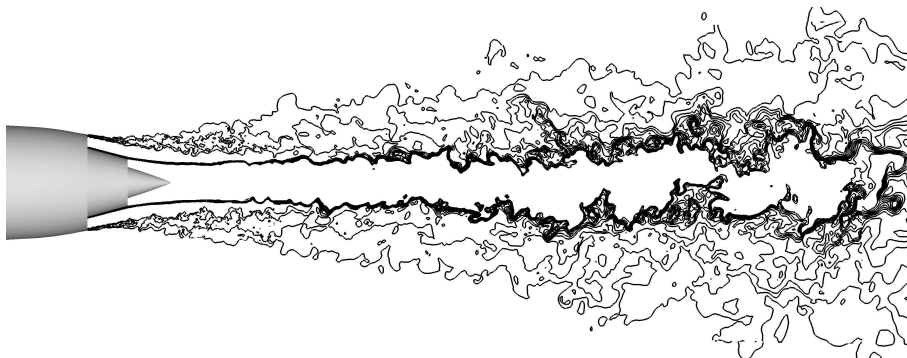


Figure 13. Instantaneous entropy contours (*case 2*)

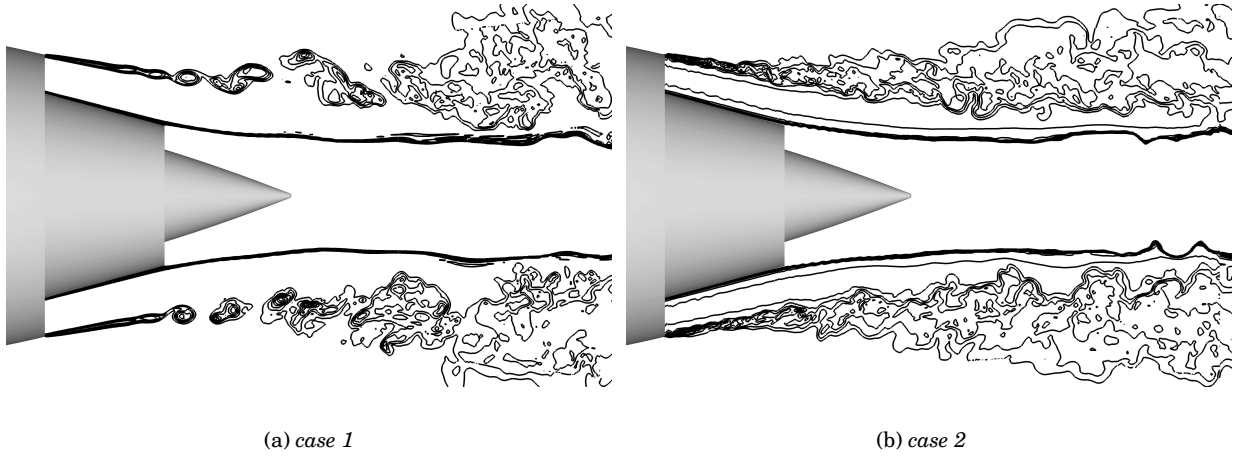


Figure 14. Instantaneous contours of entropy near the nozzle walls.

layer breaks up at an earlier stage in *case 2* than in *case 1*. Figures (16(a)) and (16(b)) again shows the differences in the initial part of the outer shear layer in the two cases. The initial mixing process is enhanced in *case 2*, which leads to that smaller flow structures are generated close to the nozzle lip than in *case 1*.

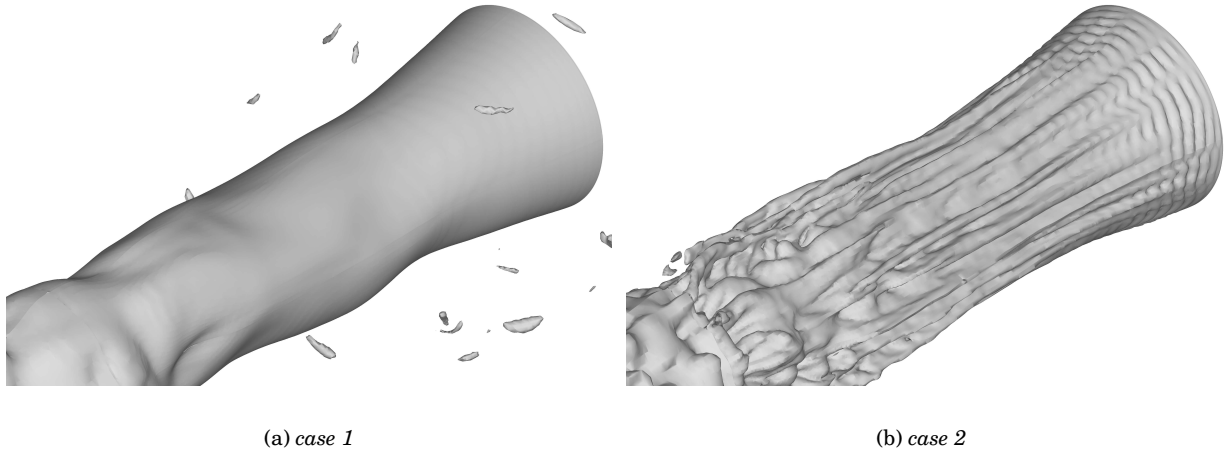


Figure 15. Density iso surfaces showing flow structures in the shear layer between the primary and secondary stream.

B. Time-averaged Flowfield

This section presents profiles of time-averaged flow quantities. Data have been obtained along three axial lines and ten radial lines in the near-nozzle region. The axial lines are situated at the centerline, $r = 0$, and along the nozzle lip lines, i.e. $r = D_p/2$ and $r = D_s/2$. The axial location of the first radial profile is $x = 0.018D_s$ and the distance between each radial profile is $\Delta x = 0.043D_s$. All profiles

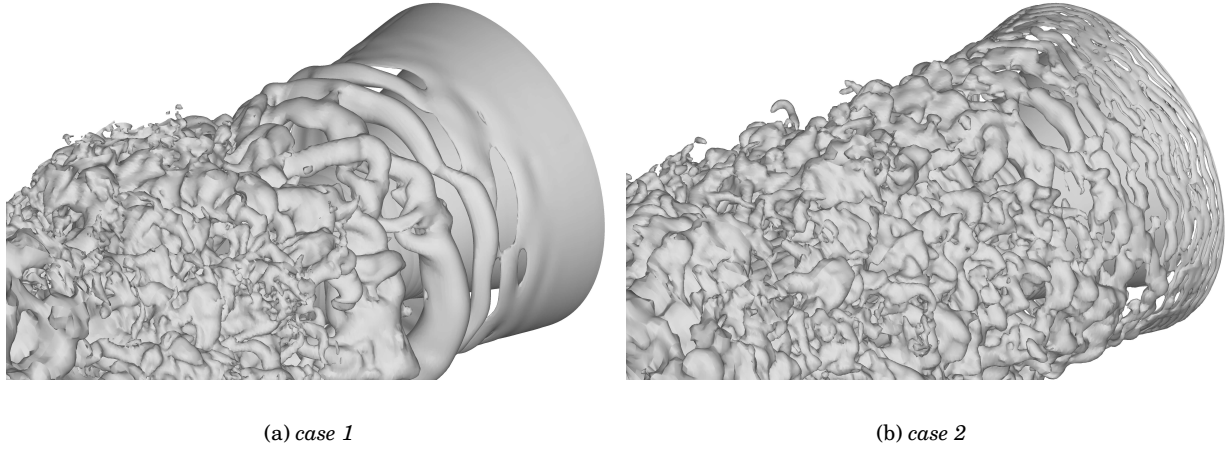


Figure 16. Density iso surfaces showing flow structures in the shear layer between the secondary stream and the stagnant surrounding fluid.

except the one extracted along the centerline have been obtained using data averaged in both time and in the azimuthal direction so as to improve the statistical convergence. It should be noted that the axial extent of the subset of the computational domain for which the databases of samples for the two simulations have been generated does not reach the as far downstream of the nozzle exit as to the end of the potential core. Therefore, no information on the predicted potential core lengths can be extracted from the time-averaged data.

Figure (17) shows axial and radial profiles of axial velocity. As can be seen from Figure (17(a)), there are some non-negligible differences in the development of axial velocity in the shear layer regions and along centerline in the two simulations. For example, *case 1* shows a longer low velocity region along the centerline downstream of the center body. For the radial profiles, however, only small deviations can be identified, see Figure (17(b)). For the radial profiles obtained close to the nozzle, the effects of the two shear layers are clearly visible. For the profile furthest downstream these effects are almost gone and the profile looks more like a single-stream jet profile. Figure (18) shows profiles of u_{rms} . In these profiles there are significant differences between the two cases. The turbulence intensity is higher close to the nozzle in *case 2* than in the *case 1* simulation in which u_{rms} is almost zero at the nozzle lips, see Figure (18(a)). At locations further downstream the intensities are higher in *case 1* than in *case 2*. The same trend can be seen in Figures (19–20), which show resolved turbulence intensity and $u'v'$ -correlation, respectively. These results are consistent with the trends seen in the instantaneous flowfield images presented in the previous section, i.e. the jet obtained in the *case 2* simulation starts to mix immediately as the jet exits the nozzle whereas the shear layers in *case 1* develops slower. However, once the mixing is initiated the mixing process is more violent in *case 1* and thus the resolved fluctuations reach higher levels.

C. Two-point Space-time Correlations

Two-point space-time correlations were obtained for the axial velocity component in a few shear layer locations along the nozzle lip line of the outer nozzle. The two-point space-time correlation of axial velocity for a certain spatial separation, ξ , and separation in time, τ , is given by

$$\mathcal{R}_{uu}(\mathbf{x}, \xi, \tau) = \frac{\langle u'(\mathbf{x}, t) u'(\mathbf{x} + \xi, t + \tau) \rangle_{t\theta}}{\sqrt{\langle u'^2(\mathbf{x}) \rangle_{t\theta}} \sqrt{\langle u'^2(\mathbf{x} + \xi) \rangle_{t\theta}}} \quad (19)$$

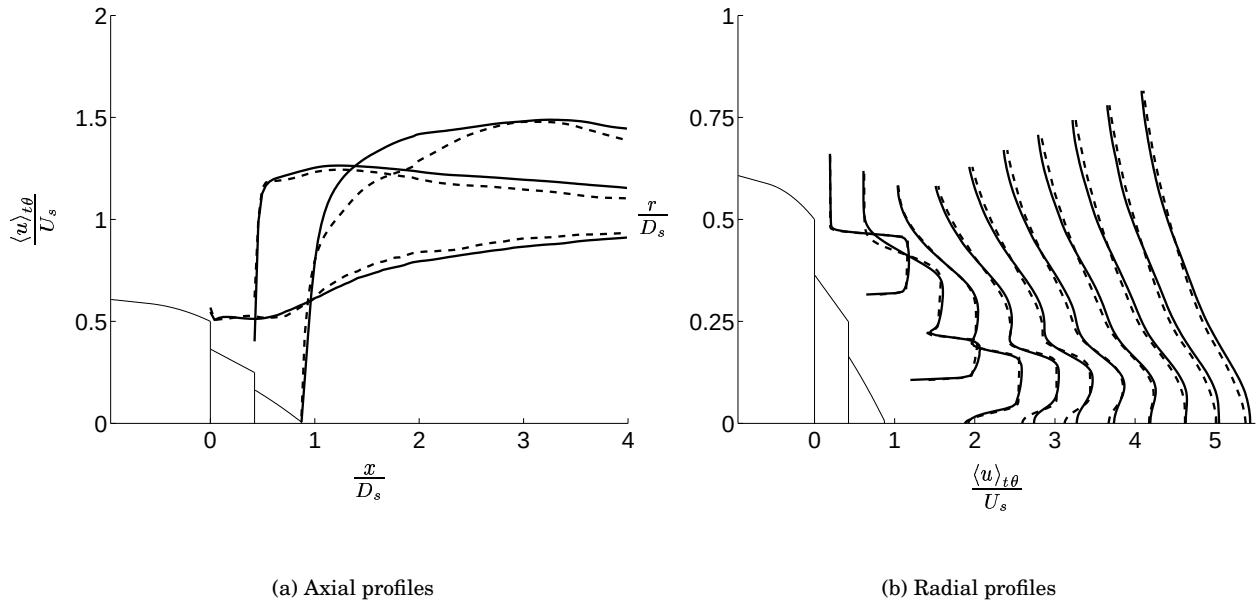


Figure 17. Axial and radial profiles of axial velocity. The profiles have been staggered according to their radial and axial location, respectively. Dashed lines denote *case 1* and solid lines denote *case 2*.

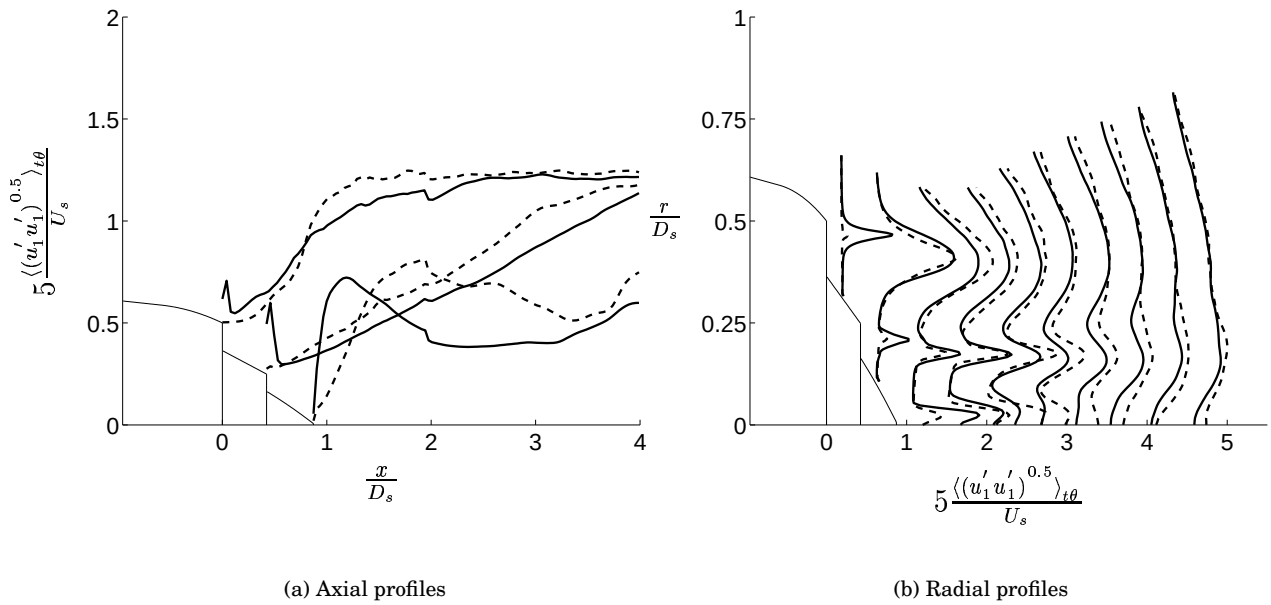


Figure 18. Axial and radial profiles of u_{rms} . For legend see Figure (17).

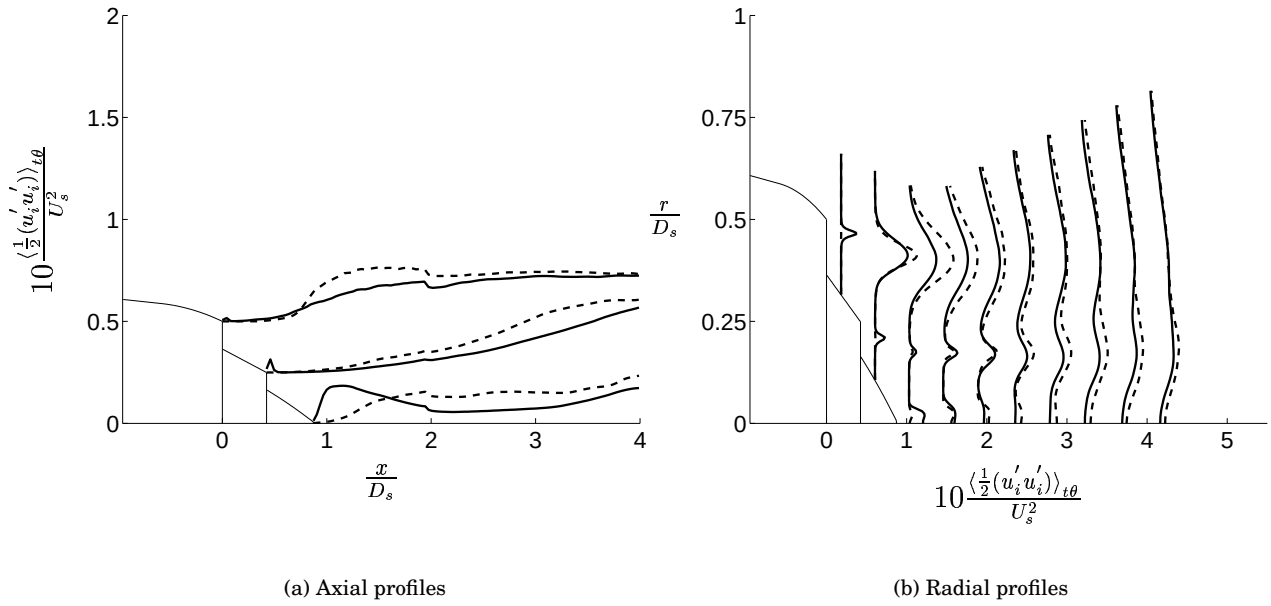


Figure 19. Axial and radial profiles of resolved turbulence kinetic energy. For legend see Figure (17).

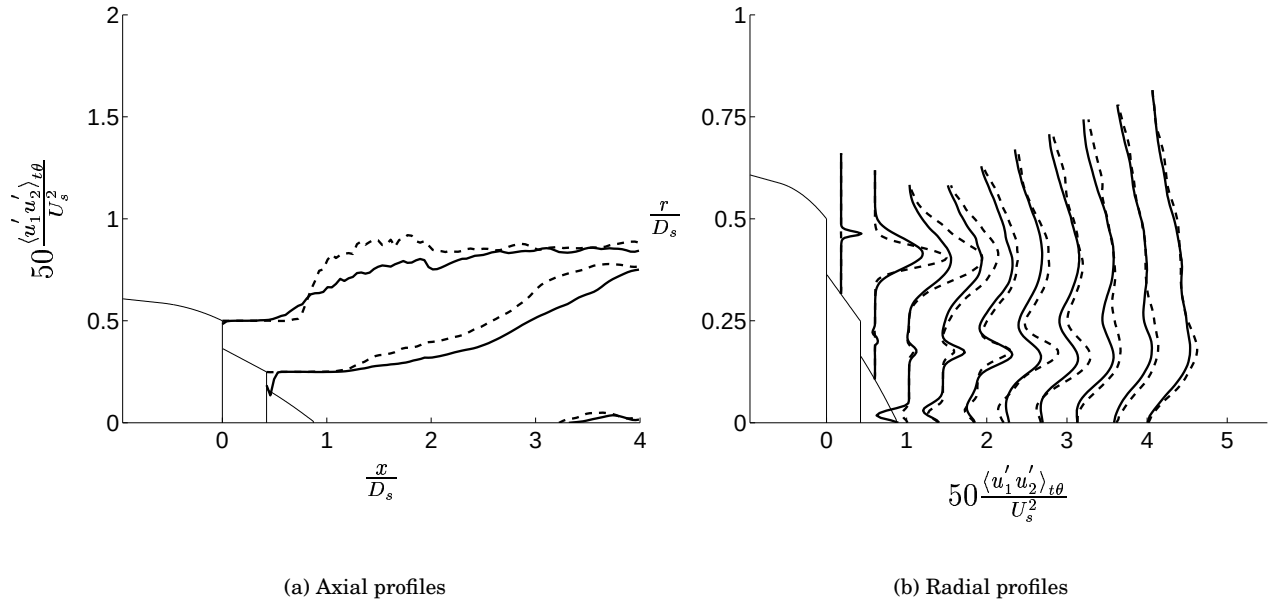


Figure 20. Axial and radial profiles of $u'v'$. For legend see Figure (17).

where u' denotes resolved fluctuation of axial velocity and $\mathbf{x}$ is the position in the flowfield where the two-point correlation is evaluated. Figure (21) compares correlations obtained in the outer shear layer along the secondary nozzle lip line at an axial location corresponding to the downstream end of the center body. The spatial separation, ξ , is downstream in the axial direction. In total 26 correlation curves are represented in each of the figures. Each consecutive curve represents an increase of the spatial separation with $2.0 [mm]$. As can be seen in the figures the correlations are wider in *case 2* than in *case 1*. This indicates that the turbulent flow in *case 2* is closer to homogeneous turbulence than the flow in the *case 1* simulation and that turbulence scales are larger in *case 2* than in *case 1*.

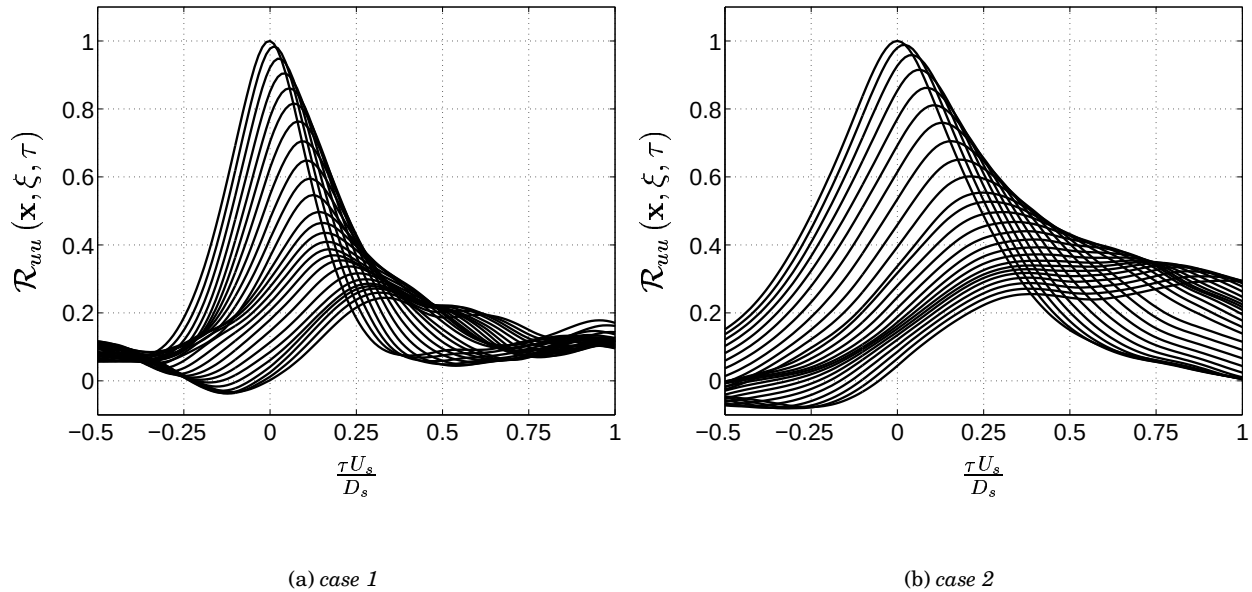
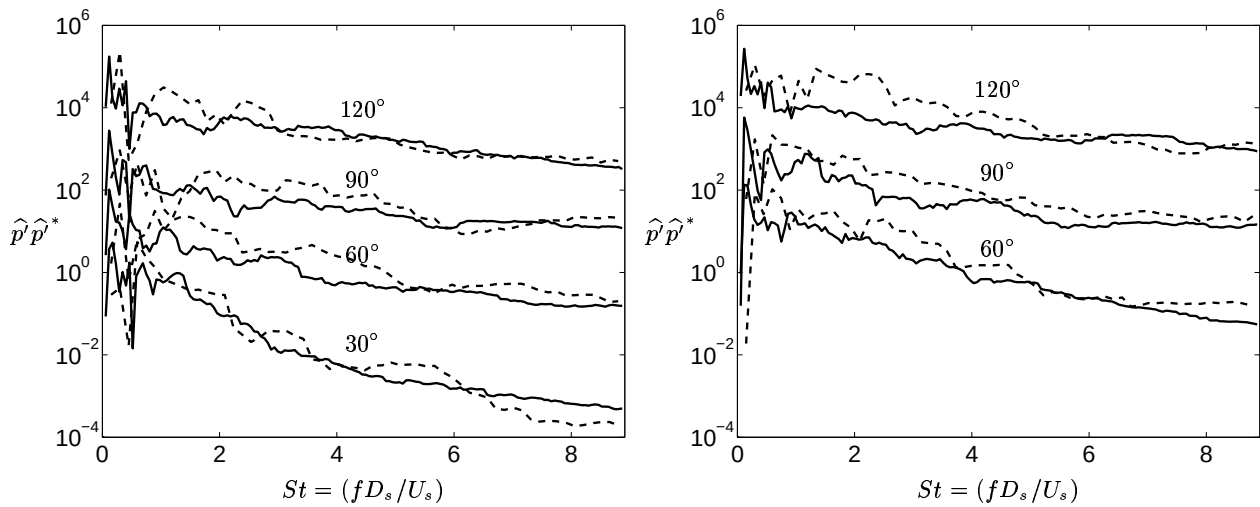


Figure 21. Two-point space-time correlations obtained in the outer shear layer at an axial location corresponding to the tip of the center body, $\mathbf{x} = (x = 0.25, r = 0.5D_s, 0)$.

D. Far-field Sound Pressure Levels

This section gives a presentation of predicted far-field sound pressure levels (SPL) and spectra of far-field pressure signals for *case 1* and *case 2*. The far-field pressure signals were obtained using Kirchhoff surface integration. Sound pressure levels are presented in 90 observer locations. These observers are situated on two arcs, one centered on the centerline at the exit plane of the outer nozzle and one centered $1.5m$ downstream of the center of the first arc, see Figure (2). The center of the second arc is approximately where the potential core ends in the RANS computations made for the same nozzle geometry and flow properties. On each of the two arcs there are 45 observer locations equally spaced between 20° to 130° from the x -axis for *arc 1* and from 30° to 135° degrees from the x -axis for *arc 2*. Figure (22) shows pressure signal power spectra for *case 1* and *case 2*. Spectra obtained for four observer locations on the arc centered at the nozzle exit (*arc 1*) are presented in Figure (22(a)) and three observers located on the second arc (*arc 2*) are represented in Figure (22(b)). Note that the pressure signal time series obtained for *case 1* is significantly shorter than the one obtained for *case 2*, which results in more uneven spectra in this case. The SPL presented in Figure (23) were obtained by integration of the power spectra from $St = 0.05$ up to a Strouhal number of 8.912, which corresponds to

a frequency of 10 [kHz]. The figures show that the SPL is for almost all observer locations significantly higher in *case 1* than in *case 2*. This is probably an effect of the high levels of turbulence kinetic energy found in the initial mixing region in *case 1*. The fact that the sound pressure levels in *case 1* are higher is most noticeable for the forward arc observers, which is natural since the forward arc is more affected by sound generated by small scale structures in the shear layers whereas the sound radiated to the rear arc observers is mainly generated by large structures in the region where the potential core ends. Consequently, the SPL peak is almost the same in the two cases but since the levels in the forward arc are higher in *case 1* than in *case 2*, the sound directivity is not the same. There are at the moment no experimental data available for validation of the results, which means that there is no evidence of that the SPL obtained for *case 2* are more correct than the ones obtained for *case 1*. Nevertheless, it is believed that the flow obtained in the second simulation is more physically correct and that the corresponding predicted SPL therefore are more representative for the current nozzle/jet configuration. In either case, these results show the importance of predicting the flow in the initial flow region accurately in order to capture the acoustic signature of the jet.



(a) arc 1

(b) arc 2

Figure 22. Power spectra of far-field pressure signal for a few observer locations on the two arcs ($r = 30D_s$). The spectra are band-pass filtered in third octave bands and have been staggered by multiplying the amplitude by a factor 10^n , where $n = \left(\frac{\theta - 30}{30}\right)$, θ being the angle from the jet axis, see Figure (2). Dashed lines denote *case 1* and solid lines denote *case 2*. $\hat{p}'$ denotes the Fourier transform of the pressure fluctuation and $\hat{p}'^*$ its conjugate.

VII. Concluding Remarks

A high-subsonic coaxial jet has been simulated using large-eddy simulation. Two simulations have been made referred to in the paper as *case 1* and *case 2*, respectively. In the first simulation it was observed that the levels of mixing in the initial part of the outer shear layer were very low. This led to a rather violent mixing process once the shear layer broke up and hence the levels of turbulence kinetic energy were rather high in this region. Furthermore, it was found that the inner shear layer was very

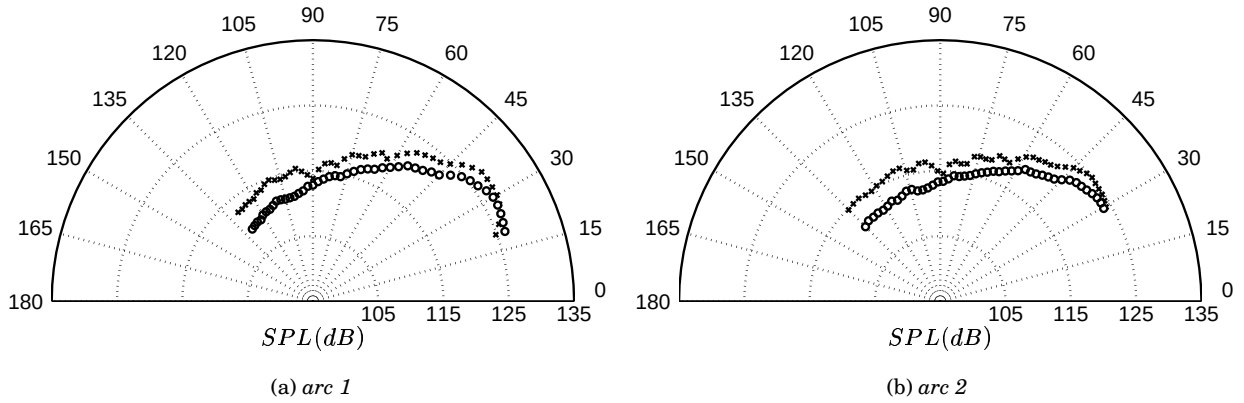


Figure 23. Sound pressure levels in observer locations situated on a $30 D_s$ arc centered at the exit plane of the secondary nozzle (a) and on an arc centered at a location 1.5m downstream of the previous location (b), see Figure (2). Predicted SPL are compared for *case 1* (x-marks) and *case 2* (circles).

thin and stable. Because of these observations a few changes were made in the simulation set-up. The subgrid-scale filter width, Δ , was decreased from $\Delta = (\Delta_1 \Delta_2 \Delta_3)^{1/3}$ to $\Delta = \min(\Delta_1, \Delta_2, \Delta_3)$ with the objective of decreasing the SGS dissipation in the regions close to the nozzle and hence improving the resolved mixing in the outer shear layer. Furthermore, the accuracy of the scheme was improved by replacing the artificial selective damping needed to obtain stability in the simulation of *case 1* to a TVD limiter for density in *case 2*. Owing to a decrease in numerical dissipation this further improved the mixing in the outer shear layer. Moreover, the shear layer between the core flow and the bypass flow became more unstable with this set-up. In both simulations total enthalpies and total pressures were specified at the nozzle inlet boundaries. However, in *case 1* constant values were used whereas in *case 2* the total pressures were specified as inlet profiles, which gave thicker boundary layers in the latter case.

For both simulations approximately 2.0×10^7 nodes were used for discretization of the calculation domain. In order to keep the number of cells down, the computational domain was divided into three regions: a well resolved near wall LES region, a medium-resolution LES region optimized for propagation of acoustic waves, and a coarse LES region. Over the interfaces between these regions, the number of cells is increased by a factor two in each direction. Special treatment of the interfaces between the regions based on cell reconstruction is utilized in order to minimize undesirable numerical errors.

The radial extent of the computational domain increases downstream such that the flow in the outer boundary region is sufficiently far from the hydrodynamic jet region and thus can be assumed to be irrotational and axisymmetric. Hence, the flow outside the three-dimensional computational domain can be represented by a less expensive two-dimensional axisymmetric calculation. Using this kind of boundary treatment enables the use of a rather narrow LES domain. Since the outer boundary of the 2D region is rather far from the jet the flow is not as affected of the boundary velocities specified at the boundary as if they were to be specified at the boundaries of the 3D domain. The interface between the full 3D LES region and the 2D region is based on azimuthally averaged quantities and acts as an absorbing boundary condition on both sides of the interface.

It should be noted that the simulations were made without increasing the viscosity or scaling down the geometry, which means that, although 2.0×10^7 nodes were used, it is a rather coarse LES. Wall functions were thus used to represent the near nozzle flow where needed.

High levels of turbulence kinetic energy found in the outer shear layer in *case 1* resulted in signifi-

cantly higher predicted sound pressure levels in *case 1* than in *case 2*. This result shows that changes in the initial mixing process have pronounced effects on the radiated sound and that it is thus important to capture the physics in the initial jet region to be able to predict the sound directivity. These effects are most noticeable for the forward arc observers. The peak sound pressure level was the same for both cases, which indicates that the initial mixing process is not as important for the rear arc observers as for the forward arc observers.

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